Control of Chaotic Systems using Harmonic Control Arrays

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***Abstract*— Recently, it has been demonstrated that the Harmonic Control Array (HCA) approach provides an effective control strategy for systems with periodic references or disturbances. In order to get zero steady-state error, the harmonic correction algorithm alters the complex levels of the system input's harmonic components appropriately. The signals, parameters, and gains involved in the system are complex valued. An efficient MATLAB/Simulink modeling of HCAs is used to simulate the control system. Besides linear systems, various nonlinear systems with periodic references or disturbances can also be effectively controlled using HCA. In this paper, HCA performance on chaotic systems is demonstrated. Especially we consider the famous Chua's circuit and Recurrent Multilayer Perceptrons (RMLPs).**

Keywords—Harmonic control arrays, chaos, Chua's circuit, Recurrent Multilayer Perceptrons

I. INTRODUCTION

Besides theoretical investigations and research, chaotic systems like Chua's circuits, Lur'e systems, Lorenz systems, recurrent multilayer perceptrons (RMLPs) and neural networks have various technological applications. One of these applications is on secure communication. Pecora and Carroll [1] proposed a drive-response system for achieving the synchronization of chaos systems. Many different kind of control methods were proposed to control chaotic systems for this purpose: linear, nonlinear, robust, adaptive, sliding mode, Lyapunov functions based control, reinforcement learning, impulsive control, event-triggered control (see [2]–[10], and references therein). In this paper, we investigate the use and performance of Harmonic Control Arrays for both stabilization and perfect periodic reference tracking of chaotic systems: recurrent multilayer perceptrons (RMLPs) and Chua's circuit.

The Harmonic Control Arrays (HCA) method is developed to control systems containing periodic reference or disturbance signals [11]–[16]. The HCA structure provides error-free periodic reference tracking at the system output by automatically configuring the periodic control signal that is given to the input of the system by compensating for periodic disturbances. HCA uses the running Fourier series integral operating in a complex structure to efficiently obtain the harmonic components of the real-time error signal in the structure of the feedback system. Each harmonic signal component is controlled individually by controllers operating in real time. Then, a real-value control signal in the time domain is obtained from the complex control vector and applied to the system input.

The proposed method is based on processing the complex harmonic frequency components of the error signal separately, instead of directly processing the real-time error signal in a classical control. In this way, it becomes possible to use separate controllers for each harmonic component. Including the DC component, one-to-one compensation of as many harmonics as the processor capacity allows can be handled.

In systems where the reference or disturbance is periodic, the harmonic components of the error signal turn into fixed complex value signals. In its simplest form, this allows the use of integral control to eliminate steady-state errors. For this reason, PI type control is preferred generally. With the help of integral controllers with complex gain that operate separately for each harmonic signal, the complex steady state error in each harmonic component is vanished and the output can successfully follow the periodic reference signal. This method makes it possible to run separate controllers for each harmonic component instead of a single controller. The term Harmonic Control Array was derived from this perspective.

The necessary block diagrams are implemented in the Matlab/Simulink environment for HCA, thus enabling the method to be easily tested on various systems [14]. It has been shown that the HCA method can produce more robust results than Internal Model Control. HCA is compared with PID and MPC control and shown to be performing successfully [15]. For nonlinear systems, on the other hand, HCA is shown to be able to construct the nonlinear control action for a pure sinusoidal reference signal tracking [16]. Details regarding Harmonic Control Arrays applications can be found from the provided references.

In the rest of the paper, we first give brief information about HCA in the next section. Then, we numerically apply HCA to two chaotic system examples using various parameters, reference and disturbance signals.

II. HARMONIC CONTROL ARRAYS

Consider a typical unit feedback system with the error signal $e(t)$ indicating the difference between the reference input $r(t)$ and the system output $y(t)$, and the control signal $u(t)$ applied to the system input. Let's assume that the fundamental period of the periodic reference and disturbance signal is T. Let's take the angular frequency as $\omega = 2\pi/T$. Let H be the highest harmonic number we will consider for control. HCA consists of three main components summarized below.

Harmonic Disperser: Using a running Fourier series integral, harmonic components can be obtained from a time domain error signal. Here, at each transaction time t, the last period slice of the incoming signal is taken into account. Therefore, changes in the levels of harmonic signals are constantly monitored as quickly as possible. hth harmonic signal (distribution) is obtained as follows:

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$$< e >_h(t) = \frac{1}{T}\int_{t-T}^{T} e(\tau) e^{-jh\omega t} d\tau \quad (1)$$

Combining these distributed signals in vector form, we obtain the dispersion of e as follows:

$$< e > = \begin{bmatrix} < e >_0 \\ < e >_1 \\ \vdots \\ < e >_H \end{bmatrix} \quad (2)$$

Since $e(t)$ is assumed to be a real-valued signal, there is no need to calculate negative harmonics here. If the signal e(t) has period T, $\langle e \rangle$ will have a complex constant vector value over time. On the other hand, the discrete time version of (1) can be given as:

$$< e >_h [n] = \frac{1}{N} \sum_{k=n-N+1}^{n} e[k] e^{-j2\pi h \frac{k}{N}} \quad (3)$$

Here $N = T/T_s$ is an integer and is the number of samples in the main period, and T_s is the sampling time. N/H must be high enough so that even the highest harmonic signal is well represented. Especially when N is high, the computational burden of (3) may be too high. In this case, we can use the following equivalent calculation for the discrete time disperser:

$$< e_h > [n] = < e >_h [n] + (e[n] - e[n-N]) e^{-\frac{j2\pi hn}{N}} / N \quad (4)$$

Once the exponential terms have been precomputed, (4) requires only one complex multiplication and a buffer of N real numbers. So, it is quite convenient in terms of real-time computation.

Harmonic PI Controller: The HCA internal controller treats each harmonic signal received from the disperser individually and attempts to build up the control signal harmonics so that the steady-state error approaches zero asymptotically and as quickly as possible. Normally, to achieve this, integral controllers are needed for each harmonic signal so that even when the error reaches zero, the integral outputs can produce appropriate complex levels to be injected into the system input. To speed up the transient response, proportional controllers can also be used.

Many alternative control techniques can be used in the internal structure of the HCA instead of the PI controllers. In our previous work, the PI control structure was preferred in the current paper because the complex PI controllers were relatively simple in structure and allowed error-free periodic reference monitoring even in nonlinear systems. Therefore, the following PI control is used to account for the dispersed components of the system input:

$$< u > (t) = K_p < e > (t) + K_i \int_0^t < e > (\tau) d\tau \quad (5)$$

Here, K_p and K_i are complex valued proportional and integral gain matrices. Off-diagonal terms in these matrices can be useful for nonlinear systems for harmonics to affect each other, but when not used, the matrices can be chosen as diagonal matrices. In this case, each harmonic error signal will only affect the corresponding control harmonic signal. The discrete-time version of (5) can be used as follows:

$$< u > [n] = K_P < e > [n] + K_i T_s \sum_{k=0}^{n} < e > [k] \quad (6)$$

Harmonic Assembler: The harmonic assembler recombines the harmonic components obtained from the HCA internal controller to form the real-time control signal. Assuming that $e(t)$ and $u(t)$ are real-valued signals, we can use Fourier series synthesis to generate the control signal:

$$< u > (t) = < u_0 > (t) + 2Re\left\{\sum_{h=1}^{H} < u >_h (t) e^{jh\omega t}\right\} \quad (7)$$

Similarly, the discrete controller representation can be obtained as:

$$< u > [n] = < u_0 > [n] + 2Re\left\{\sum_{h=1}^{H} < u_h > [n] e^{\frac{j2\pi hn}{N}}\right\} \quad (8)$$

Determining the number of harmonics, H, is at the discretion of the designer to remain within the hardware capabilities. Increasing the number of harmonics gives better results, especially in non-linear loads, at the cost of using more processing power.

III. APPLICATION TO CHAOTIC SYSTEMS

Let us use the following discrete-time system representation to consider nonlinear or chaotic systems:

$$\begin{cases} x(k+1) = f(x(k)) + B(u(k-N_d) + d(k)) \\ y(k) = Cx(k) \end{cases} \quad (9)$$

where $x \in \mathbb{R}^n$ is the state vector, $f()$ is the nonlinear state transition function, $B \in \mathbb{R}^{n \times r}$ is the input matrix, $u \in \mathbb{R}^r$ is the input vector, $N_d \in \mathbb{N}$ is the sample delay in the input, $d \in \mathbb{R}^r$ is the disturbance acting on the input of the system, $y \in \mathbb{R}^m$ is the output vector, $C \in \mathbb{R}^{m \times n}$ is the output matrix. We can construct a model of the system (9) in Simulink as shown in Fig. 1.

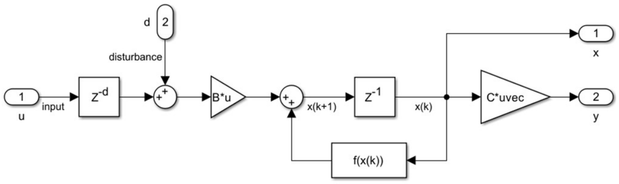

Fig. 1. A nonlinear discrete-time system model.

A discrete-time Harmonic Control Arrays feedback system is constructed in Simulink environment as shown in Fig. 2 for controlling the chaotic system in Fig. 1. The details of the construction of the HCA blocks can be found in [14].

A. Recurrent Multilayer Perceptrons

Let us consider the discrete-time chaotic RMLP [2] as in the form of (9) where

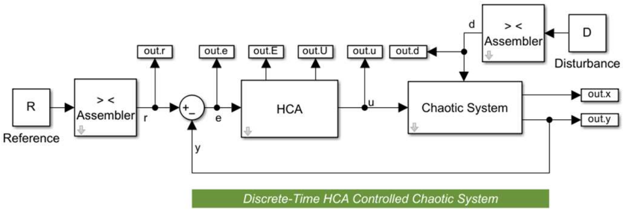

Fig. 2. Simulink model of a Harmonic Control Arrays controlled chaotic system.

$$f(x) = W tanh(Vx), \tag{10}$$

$$W = \begin{bmatrix} 0.9690 & 0.6967 & 0.2985 \\ -0.7473 & 3.2069 & 0.2840 \\ -2.7960 & 0.5360 & 0.9597 \end{bmatrix},$$

$$V = \begin{bmatrix} -0.9394 & -0.0078 & -0.5210 \\ -0.6861 & -0.1108 & -0.0729 \\ 0.0879 & 0.3845 & -0.7007 \end{bmatrix},$$

$$B = [1 \quad 0 \quad 0]^T, C = [1 \quad 1 \quad 0].$$

Choosing the initial state as $x(0) = [0.2 \quad -0.2 \quad 0.14]^T$, we obtain a chaotic trajectory of the system when there is no control as shown in Fig. 3.

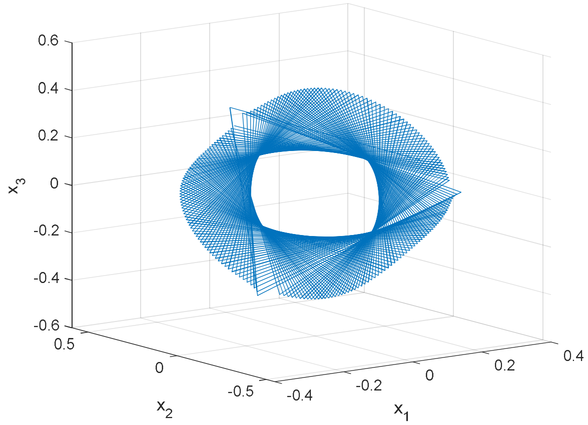

Fig. 3. A chotic trajectory of the RMLP system for 200 samples.

To obtain a reference signal of $r(k) = \cos\left(\frac{\pi}{3}k\right)$ at the output of the chaotic system (10) we construct HCA parameters as

$$N = 6,\ H = 3,\ R = [0 \quad 0.5 \quad 0 \quad 0]^T.$$

We choose PI gains as shown below:

$$K_p = [0 \quad 0 \quad 0 \quad 0]^T,$$
$$K_i = [-0.3 \quad -0.15j \quad 0.01 + 0.02j \quad -0.02]^T.$$

When there is no disturbance on the system, and the HCA started acting at the sample $k = 100$, the output and the error between the output and the reference are shown in Fig. 4. As seen from the results, chaotic behavior is replaced by a periodic sinusoidal signal in about a few periods.

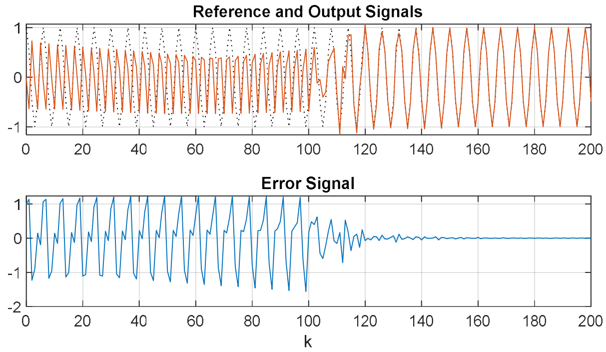

Fig. 4. The output and the error signals of the RMLP system.

To obtain the reference sinusoidal signal at the system output $y(k)$, the state trajectory of the system transforms to a periodic orbit as shown in Figs. 5 and 6.

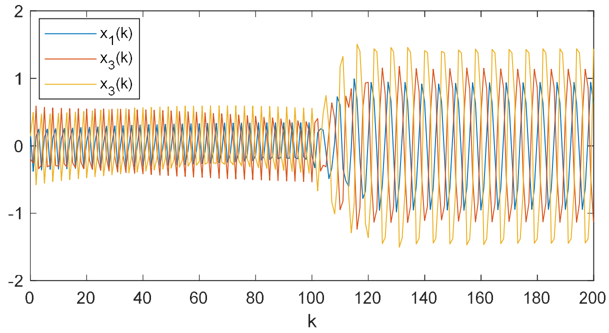

Fig. 5. States of the system before and after the HCA control.

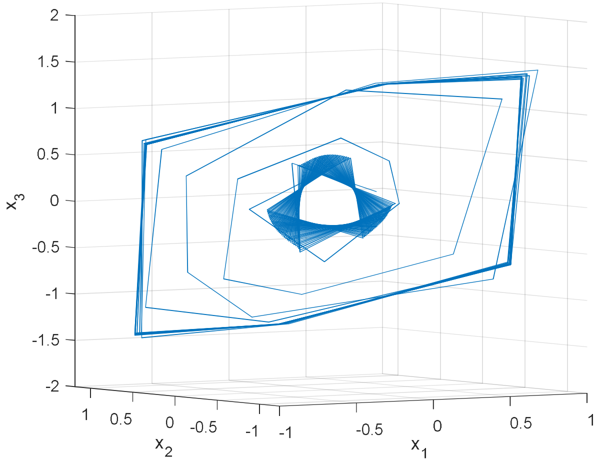

Fig. 6. Transforming from a chaotic to a periodic orbit.

Here, the HCA gradually constructs the necessary periodic control signal as shown in Fig. 7. As seen from the figure, the harmonic components are used in the control signal to obtain the pure sinusoidal wave at the system output.

Now, using the same HCA controller, let us try its performance for some other reference and disturbance signals. Choose:

$$R = [0.3 \quad 0.1 \quad -0.2 \quad 0.2]^T,$$
$$D = [0.1 \quad 0.2j \quad -0.1 \quad -0.1]^T.$$

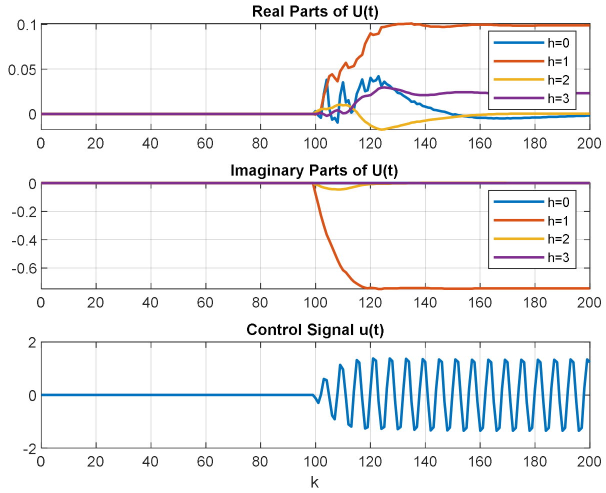

Fig. 7. Constructing of the control signal for RMLP sinusoidal output.

Corresponding reference and disturbance signals are shown in Fig. 8 with the system output and the error signals. As we see, although there is a disturbance, the required reference signal is obtained at the system output without any steady state errors in about 80 samples.

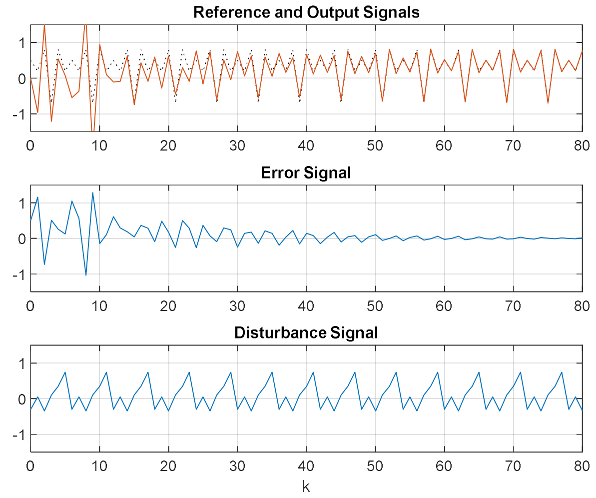

Fig. 8. HCA control perfornance of RMLP system with the dsiturbance.

To overcome the disturbance signal, and obtain the necessary output signal, the state signals must go to a certain trajectory as shown in Fig. 9. During this time, the dispersion signals of the output and the error are shown in Fig. 10. As depicted, the output dispersions tend to the necessary values as required by the reference, and the error magnitudes for each harmonics decay to zero.

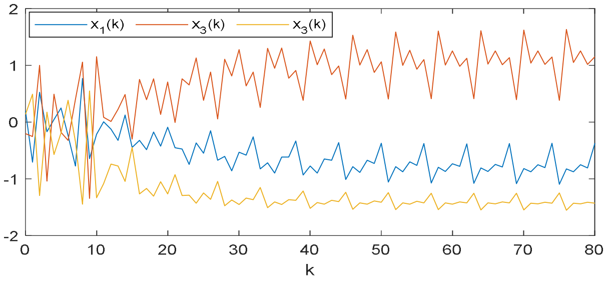

Fig. 9. State signals for RMLP system with the disturbance.

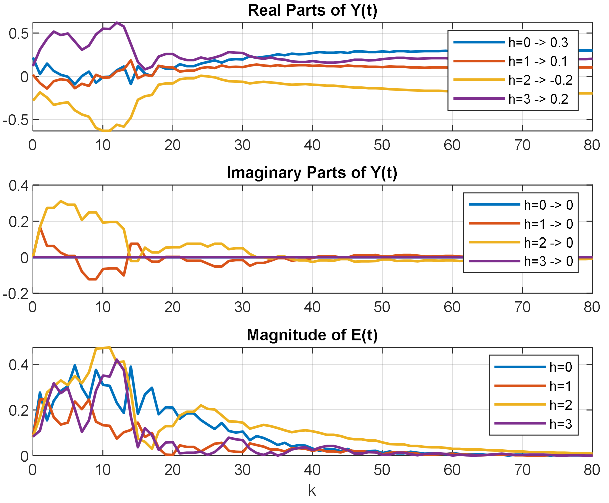

Fig. 10. The dispersions of the output and error for RMLP system.

The constructed real time control signal through the output dispersions of the HCA is shown in Fig. 11. This signal successfully controls the system despite the disturbance and achieves zero steady state error.

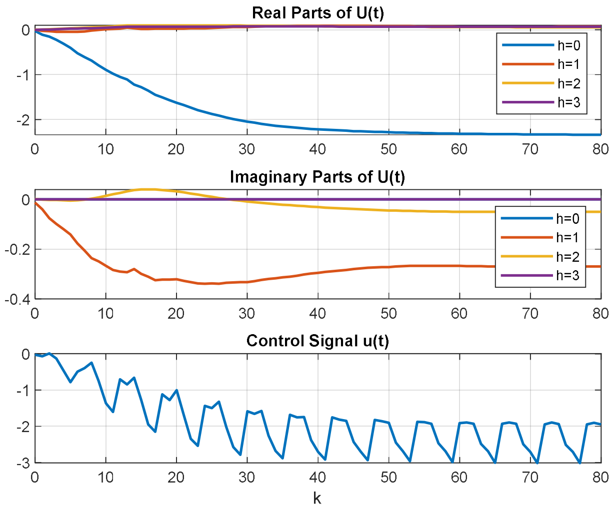

Fig. 11. Construction of the stabilizing control signal for RMLP system.

To test if the HCA could stabilize the RMLP system with a delayed input, and achieve the zero steady state error, we can easily modify the existing PI gains using the inverse delay dynamics compensating the delay frequency response for each harmonic, as show below:

$$\widetilde{K}_i(h) = K_i(h)e^{2\pi jhN_d/N}, \quad h = 0,\cdots,H. \qquad (11)$$

If the resulting transient response for these new gains is stable and fast enough, this simple PI gain adjustment will provide a similar steady state performance, i.e. resulting zero steady state error and perfect tracking. Using these adjusted gains for a two-sample delay, $N_d = 2$, the HCA provides satisfactory results for a sinusoidal reference despite the added disturbance as shown in Fig. 12.

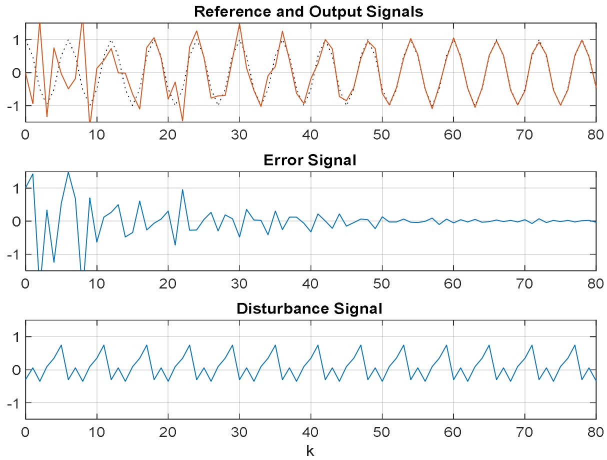

Fig. 12. The performance of HCA control of the delayed RMLP system.

B. *Chua's Circuit*

Using (9), a discrete-time model of Chua's circuit [3]-[7] can be given as follows:

$$f(x) = x + T_s \begin{bmatrix} \alpha(-x_1 - g(x_1) + x_2) \\ x_1 - x_2 + x_3 \\ -\beta x_2 \end{bmatrix}, \qquad (12)$$

$$g(x_1) = bx_1 + \frac{1}{2}(a-b)(|x_1+1| - |x_1-1|),$$

$$B = [1 \quad 0 \quad 0]^T, C = [1 \quad 1 \quad 0].$$

where $\alpha = 9, \beta = 15, a = -1.45, b = -0.8$. The sampling time is chosen as $T_s = 0.001$ s, and the initial condition as $x(0) = [1.1 \quad 0.1 \quad 1.5]^T$. The state evolution of this system when there is no control is shown in Fig. 13 where the chaotic behavior is clearly demonstrated.

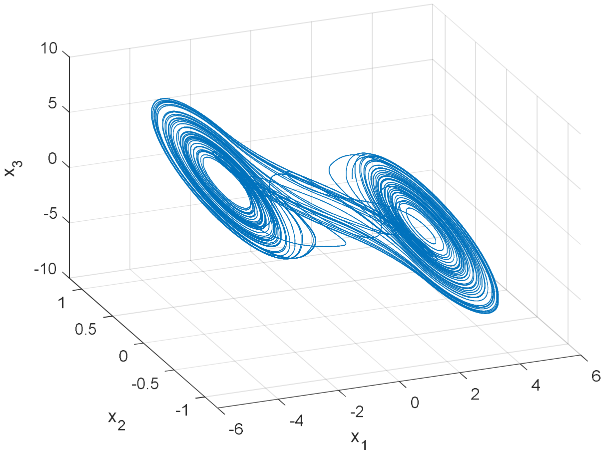

Fig. 13. A chotic trajectory of the Chua's circuit.

As required by many chaotic system applications, let us construct the HCA to drive the system output to zero equilibrium first. For this we choose:

$$T = 6T_s, \ H = 3, \ R = [0 \quad 0 \quad 0 \quad 0]^T.$$

PI gains are selected as:

$$K_p = [0.07 \quad -0.6 \quad 0 \quad 0]^T,$$
$$K_i = [5 \quad -150 \quad -150 \quad -100]^T.$$

For these choices, and applying the control after 90th second, the system is effectively stabilized as shown in Fig. 14. As seen from the close-up figures, the system output quickly vanishes in about 0.05 s. The states, on the other hand, asymptotically tend to zero in a much longer time, in about 5 s as shown in Fig. 15. However, during this time, as desired by the control performance, the system output $y(k)$ is kept zero. This is an interesting performance of the constructed HCA, since even before the states are stabilized, the required output has already vanished to zero.

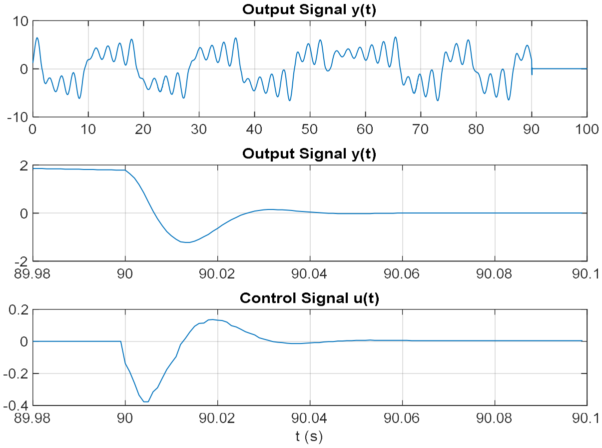

Fig. 14. Driving the system output to zero after the 90th second.

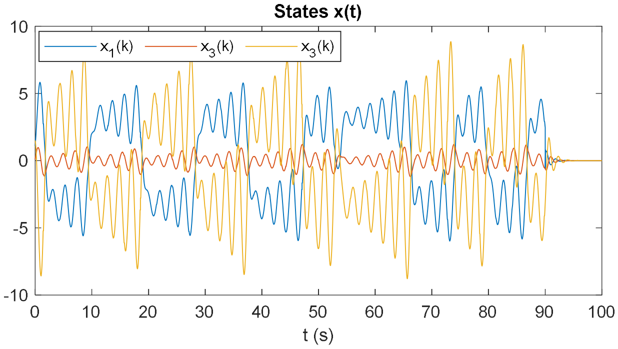

Fig. 15. State trajectories of Chua's circuit before anf after control action.

Now, instead of zero output, choose a sinusoidal reference as $r(k) = 10\cos\left(\frac{\pi}{3}k\right)$, and a random like periodic disturbance as

$$R = [0 \quad 5 \quad 0 \quad 0]^T,$$
$$D = [-0.5 \quad 1 \quad 0.5 \quad 1.5]^T.$$

Using the HCA controller with the same PI gains, the required pure sinusoidal output is obtained at the output in about 0.03 s despite the acting disturbance as seen in Fig. 16.

The dispersion of the output is shown in Fig. 17. As following the reference dispersion vector, R, the imaginary and real harmonics of the output, tend to zero, except the real main harmonic (h=1) which tends to 5. The magnitudes of the error dispersions, on the other hand, vanish to zero, ensuring perfect tracking for each harmonic signal.

The construction of the real periodic control signal out of the control signal dispersions are shown in Fig. 18. Therefore, a compensating control action for zero steady state error despite the acting disturbance signal is automatically obtained with HCA.

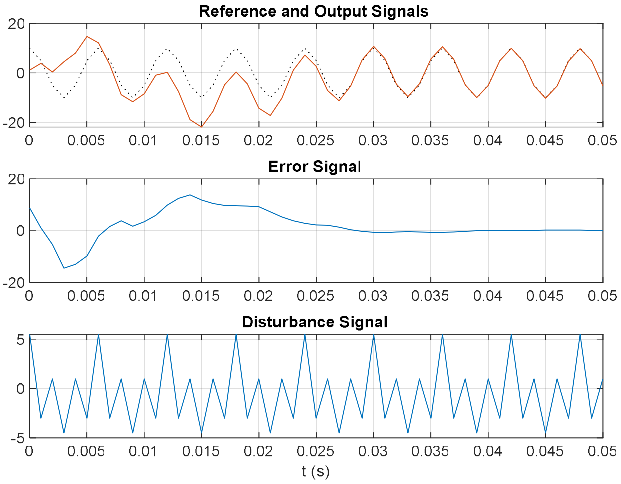

Fig. 16. Obtaining a pure sinusoidal output from Chua's circuit.

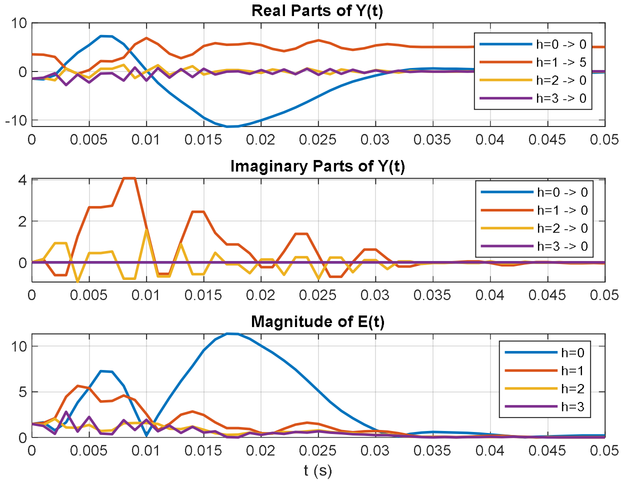

Fig. 17. The dispersions of the output and error for Chua's circuit.

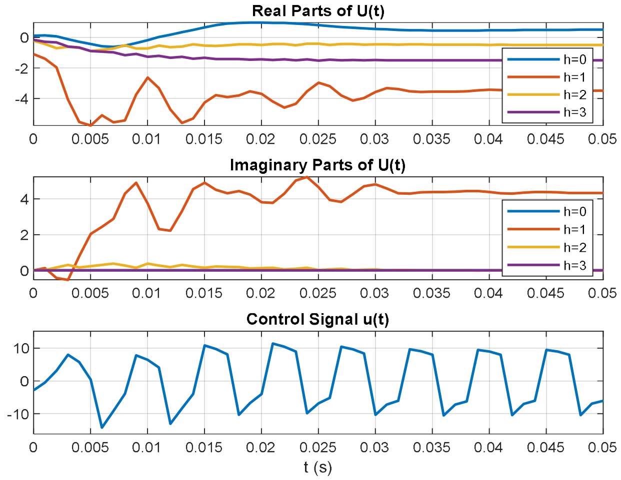

Fig. 18. Construction of the required control for Chua's circuit.

IV. CONCLUDING REMARKS

The Harmonic Control Arrays method is applied to chaotic systems in this paper. It is shown that despite the disturbance and delay, nonlinear compensation can be constructed using HCA. While keeping the stability for all harmonics, a fast transient response can be achieved, making the error between the reference and the output vanishing quickly, providing perfect tracking of a desired periodic signal. This suggests that HCA can be used as easy control design as the PID control for a nonlinear system such as a chaotic system synchronization for secure communication. In future work, an adaptive strategy can be studied for automatically adjusting the optimum complex PI gains for HCA.

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