Nexus Network Journal (2026) 28:249–270
https://doi.org/10.1007/s00004-025-00856-6

NEXUS NETWORK JOURNAL
Architecture and Mathematics

RESEARCH

A Parametric Design Method for the Wooden Turkish Triangle Plated Structure (Barkat) Model

Erdem Köymen[1] · **Muhammed Emin Akyürek**[1]

Received: 30 October 2025 / Accepted: 23 December 2025 / Published online: 19 January 2026

Abstract

Mimar Sinan's mosques are celebrated as masterpieces of central-plan mosque design, showcasing exceptional spatial composition and structural form. A previous study designed a novel model by adapting these forms using a catenary curve, folded plates, and Turkish triangles. However, that model established only a static topology. Distinguishing itself by advancing this topology into a dynamic computational framework, this research introduces a novel parametric algorithm that mathematically defines the Turkish triangle. This method allows for both the analytical classification of historical transition zones and the generation of the contemporary 'Barkat' mosque model. Using Grasshopper and Python, geometries are defined via wall segmentation/folding (n) and hoop (ring) edge (k) parameters. The analysis confirms that combinations of (n = 1–3) and (k = 4–8) yield the most balanced forms. The resulting framework offers a dual contribution: a digital reconstruction tool for architectural history and a fabrication-ready design method for folded plate structures.

Keywords Islamic architecture · Mimar Sinan · Structural form · Timber folded plate · Parametric design

Introduction

In monumental masonry architecture, achieving the geometric transition between a square infrastructure (base) and a spherical superstructure (dome) has been a persistent challenge across nearly all styles. In common practice, this transition zone between the square base and the dome is articulated using half-domes, pendentives,

✉ Erdem Köymen
erdemkoymen@yahoo.com

Muhammed Emin Akyürek
muhammed.akyurek@izu.edu.tr

[1] Department of Architecture, Istanbul Sabahattin Zaim University, Halkalı Caddesi, 34303 Istanbul, Turkey

squinches, muqarnas, and the ‘Turkish triangle’. Although ‘Turkish triangle (TT)’ (*Türküçgeni*) is a contemporary term, the form is commonly found in regions once ruled by old Turkmen authorities. In the *Usul-i Mimari-i Osmani* (Komisyon 2015), these forms were described as the planar (*müstevi*) style, whereas muqarnas was termed the ornamented (*mücevheri*) style. Due to its geometric arrangement has also been called diamond (*baklava*) or fan-shaped (*yelpaze*) ornamentation. Master builders referred to it as seven-eight (*yedi-sekiz*), referencing its visual similarity to the Arabic numerals for seven (٧) and eight (٨), which captures the essence of a triangular unit being inverted and repeated (Uluengin et al. 2016). Arseven (2017) used terms like fragmented (*münkesir*) form for the zones where this component was applied. Nowadays, the terms TT and triangular band (*üçgenli kuşak*) are commonly used.

Found across a vast geography from Anatolia to Turkestan, the TT serves not only as a transition element but also as a column base, capital, roofing, and decorative element (Fig. 1). Its most widespread and advanced use as a structure that defines architectural identity was during the Ottoman Beyliks period. In the subsequent Ottoman classical period, structures adopted smoother, less fragmented surfaces, and the TT shifted from a structural component to a primarily decorative one. The TT continues to be used in contemporary structures, much as in the classical Ottoman period. It is typically employed for decorative purposes in mosque construction, such as in the Marmara Faculty of Theology Mosque in Istanbul.

Although several studies on historical construction and restoration discuss TT (Okçuoğlu 1995; Özcan 2008; Kula Say 2011; Uluengin et al. 2016; Aktuğ Kolay 2017; Turan 2018), research on its systematic geometric definition remains limited. Cansever (2010) provided the earliest (1949) known geometric analysis through his study of Ottoman column capitals. Batur (1980, 1986) examined phased TT construction in Beylik-period buildings, while Şimşek (2010) and Morita (2017) offered typological classifications based on regional examples. However, these works are largely documentary and lack numerical or computational methods. Contemporary analytical approaches are rare; for example, Çağlar et al. (2021) generated a parametric model of the TT for a single Bursa Hacılar mosque but did not address broader variation. This indicates a critical gap for a computational framework that can generalize the geometric logic of historical typologies.

Fig. 1 Usage of the TT in historical architecture: roof, transition zone, and column capital

The study's second focus is on adapting these TT zones for contemporary construction. Aside from concrete imitations, TT has not previously been rationally addressed in a way that is systematic and true to modern materials. The underlying framework for this adaptation is the 'Barkat' model, inspired by Mimar Sinan's mosques, a Cross-Laminated Timber (CLT) structure originally developed in Chapter 4 of a doctoral thesis (Akyürek 2023) and topologically introduced in a previous study (Akyürek 2024). However, this topological model possesses a highly complex geometry and remains static, unable to adapt to specific site dynamics or functional requirements. Thus, while serving as a methodological continuation of the aforementioned studies, this research distinguishes itself by introducing a novel computational algorithm that transforms the previously static topology into a dynamic, fabrication-ready system.

Based on the identified gaps in both historical analysis (lack of systematic definition) and contemporary adaptation (static model), this study addresses two primary research questions (RQ) and corresponding hypotheses:

- RQ1: How can a systematic, computational logic be defined and classified based on the typological characteristics of (some) historical TTs to facilitate their digital reconstruction and variation?
- Hypothesis 1: A coordinate-based parametric algorithm -specifically defined by wall folding (n) and hoop edge (k) counts- can accurately classify and reproduce the geometric logic of these historical transition zones.
- RQ2: How can the static Barkat mosque model be transformed into an adaptable design system compatible with contemporary fabrication constraints?
- Hypothesis 2: Integrating the parametric TT definition with folded-plate logic will enable the generation of structurally balanced and dimensionally adaptable variations suitable for CLT construction.

By encoding historical design principles via the Rhinoceros3D/Grasshopper3D (GH) interface, the proposed method gains formal and dimensional variability. This study is the first to develop a generative system for the TT, relating it to folded plates and offering a novel tool for both contemporary design and the geometric analysis of historical buildings.

The scope of this research is limited to the geometric rationalization and parametric definition of TT within the context of Sinan's central-plan mosques and the contemporary Barkat model. The generative algorithm is designed to operate on any plan perimeter defined by straight lines lying on the horizontal (XY) plane, including rectangular, square, and polygonal boundaries. Curved plan traces remain outside the scope of the algorithm's operational capacity. While the generated forms are designed to be compatible with the fabrication logic of industrial wooden panels (e.g., plywood, CLT), detailed structural engineering analyses (e.g., Finite Element Method under dynamic loads), material stress tests, and building physics simulations (thermal/daylight) remain beyond the scope of this specific article. The study assumes standard plywood or CLT panel behavior to validate the geometric feasibility of the proposed algorithmic model.

Conceptual Background

Formation of Turkish Triangles (TT)

TT zones vary according to specific formal rules and their dimensional and proportional parameters. These variables are presented in Table 1, using examples of the transition zone from the wall to the dome. In the first example, the wall and the dome are divided at specific intervals (*n* and *k*), and these segments are covered by a series of simple, opposing planar triangles. The second example is a planar triangle version of a pendentive (spherical triangle). Here, the wall unit is singular (*n*) while the dome base is multi-segmented (*k*). The third example is formed by tiering the first. The zone typically consists of two, three, or more layers of equal height (*h*). The fourth example, a patterned version of the first, is the most common type. Here, the triangular plates are fragmented into concave or convex diamond shapes. In the fourth and fifth examples, the triangles are shown fragmented in a nested, hierarchical manner. This nesting is sometimes related to facilitating construction with smaller components, but it is mainly for aesthetic reasons. The seventh example is created by stacking the fourth one, similar to the third one. The photograph shows two distinct TTs at two levels: the lower one has four layers, and the upper one has two. Finally, the eighth example illustrates forms like pendentives, half-domes, and squinches filled with triangles in a stellate pattern. In this method, the straight and curved lines that define the form are divided into equal parts, and the resulting points (*n* and *k*) are connected. Subsequently, the resulting triangles are further divided into nested triangles based on their span or size.

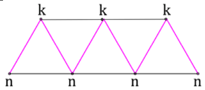
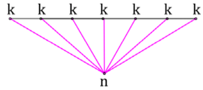
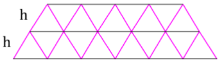
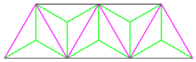
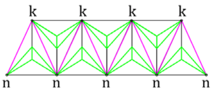
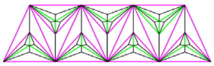
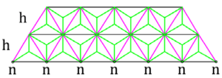
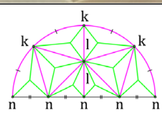
TT zones are used in various dimensions depending on their location, such as in capitals, the wall-to-dome transition, and minaret bases or balconies. The degree of shallowness is a matter of preference. In some structures, planar and diamond triangles are used side-by-side or nested within each other. The terminations at the corners can be a single line (as in the sixth example), a triangle (as in the first), or a rhombus (as in the fifth). The pyramidal triangles can be configured as convex (fifth example) or concave (sixth example).

Origins of the Barkat Model

The Barkat[1] model derives its complex geometry and relative positioning from the topology of Sinan's mosques, based on a specific conceptual background, functional

[1] 'Bark', meaning 'house' in Old Turkic and commonly used in the expression *ev-bark*, denotes a frequently visited or inhabited space (Nişanyan 2025). The modern Turkish *barınak* (shelter) derives from the same root, and historical usage extends to houses, mausoleums, and worship spaces (Useev 2016). The name *Barkat* draws on this semantic lineage: rather than referring strictly to a mosque, it designates a gathering or dwelling space adaptable to various functions. The suffix *-at*, an Arabic plural form, occasionally attached to Turkic-origin words in Ottoman and Chagatai sources, emphasizes multiplicity, expansion, and adaptability. The syllable *kat* (fold) simultaneously evokes folded-plate geometry and the TT logic central to the model. Additionally, the concept resonates with the Old Turkic verb *barı(-mak)* (to shelter; wall; hedge) and with Mediterranean terms such as *baracca/barraque/baraka*, used for single-room temporary structures and sharing both phonetic and semantic affinity. In sum, *Barkat* denotes a generative wooden folded-plate system designed for medium- and large-span structures, integrating parametric geometry and prefabrication principles.

Table 1 Classification of transition zone geometries in selected historical buildings: this inventory categorizes key configurations (TT) to establish the typological basis for the parametric abstraction

1. Planar triangle	 	2. Planar fan (yelpaze) triangle	 
3. Tiered planar triangle	 	4. Diamond (baklava) triangle (a)	 
5. Nested diamond triangle (b)	 	6. Double nested diamond triangle (c)	 
7. Tiered diamond triangle	 	8. Stellate infill triangles	 

requirements, and formal framework (see Akyürek 2023). Fundamentally, Sinan's mosques consist of a central dome on a square or rectangular base. The main dome transfers its load to the ground through four, six, or eight-column baldachins (Akyürek and Kahraman 2021). The transition zone between the dome and the rectangular substructure is achieved with curvilinear elements like half-domes, squinches, and muqarnas. In the proposed model, the relative positions (topology) found in Sinan's mosques were first schematized using a catenary model, which was identified as overlapping with Sinan's mosque sections and utilized to provide structural efficiency. Sinan's mosques were idealized as a convex form with a square base and an apex point. This ideal form was then triangulated to suit the planar nature of timber plates. These triangular plates were optimized according to the static principles of folded plates. The stylistic characteristics of the model were determined by establishing an analytical pattern between the folded plate and the TT. The dimensional variables of the model were defined by proportional relationships depending on the selection of folded timber plates. The arcaded wall substructure, the transition zone (with half-domes and squinches), and the central dome in Sinan's mosques share a topological similarity with the Barkat's folded plate wall, TT zone, and folded pyramid, respectively, representing a transformation from curvilinear to linear geometry. The resulting design model was discussed regarding its plan, section, and elevation. Thus, a topological and technological adaptation from Sinan's mosques to the present was realized (Fig. 2).

The model section was aligned with the catenary curve (Fig. 3). Here, the catenary curve was arranged to remain within the dome's walls as much as possible, and its endpoints were kept within the building's base area, just as in Sinan's mosques. If the curvilinear surfaces of the superstructure (comprising the transition zone and roof) are adapted as planar surfaces to suit the formation of timber plates, the typical section emerges, consisting of levels $h_{2,a}$, $h_{2,b}$, $h_{3,a}$, and $h_{3,b.}$ Thus, the proposed typical section benefits from the proportions of Sinan's mosques while approaching structural ideality via the catenary curve. Figure 3 illustrates the dimensional variables of the sample model, such as the width and height of the building, wall, transition zone, and roof components. Additionally, there are angular variables, such as the angles between wall plates ($\alpha_{a,b,c}$), the wall folding angle (α_k), and the wall inclination angle (α_d). The model can be formed by repeating a 1/4 corner module, or in some cases, a 1/8 module. Although wall thickness (detail x) and the minaret (which requires separate consideration) are exemplified in Fig. 3, they were not addressed in this study.

Materials and Methods

Design Modification Method

The formal structure of the Barkat consists of the wall, header, transition, and roof elements, moving from bottom to top. The roof is shaped by the number of edges (n) and the span of the hoop it forms at the apex of the transition zone. The hoop is a folded pyramid with a square, hexagonal, or octagonal base, similar to the baldachin

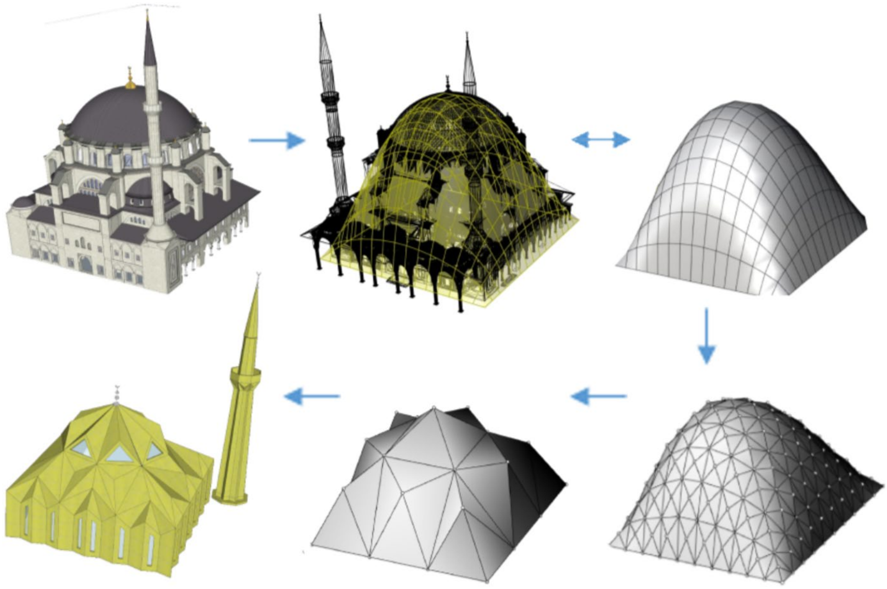


Fig. 2 The transformation of an ideal Sinan Mosque into a Barkat model: First, mosque is schematized using a catenary model, then it evolves into the Barkat through triangulation

schemes in Sinan's mosques. This part has a formation that can be encountered in various architectural styles. The triangular transition zone below the dome extends to the wall via triangular-gabled headers at the intersection (Fig. 4).

The headers, which form the intersection of the triangulated folded-plate wall, are examples of topological transformation, consistent with the model's general character. Figure 4 provides geometric drawings for five different header types (B_1–B_5), based on common arch types, illustrating segmental, round, and asymmetric variations. The infill of the header is composed of planar and diamond triangles, just like the TT (colored on elevation). The first two types (B_1, B_2) determine the size of the triangular panels forming the header while connecting the wall panels. Forming a balanced and proportional header depends on the angle of the wall panels. The third, fourth, and fifth types (B_3, B_4, B_5) are applied when rhythm and stepping are required on the façade. For example, B_3 is suitable for emphasizing an entrance; B_4 for softening an awkward corner with stepping and increasing building visibility; and B_5 for inserting rectangular joinery.

The simplest wall type (D_1) consists of two symmetrical trapezoidal plates, as in a classic folded plate. This type suits small, low-rise buildings due to its simple, single-part configuration. D_2 allows for wider structures with fewer repetitions, owing to the increased number of plates relative to the header. The plate length can be shortened or extended by changing the wall's angle (α) relative to the header. D_3 and D_4 offer a five-plate module, enabling wide doors and passages. D_5 and D_6 are dual combinations of D_1 and D_2, respectively. These types become necessary for

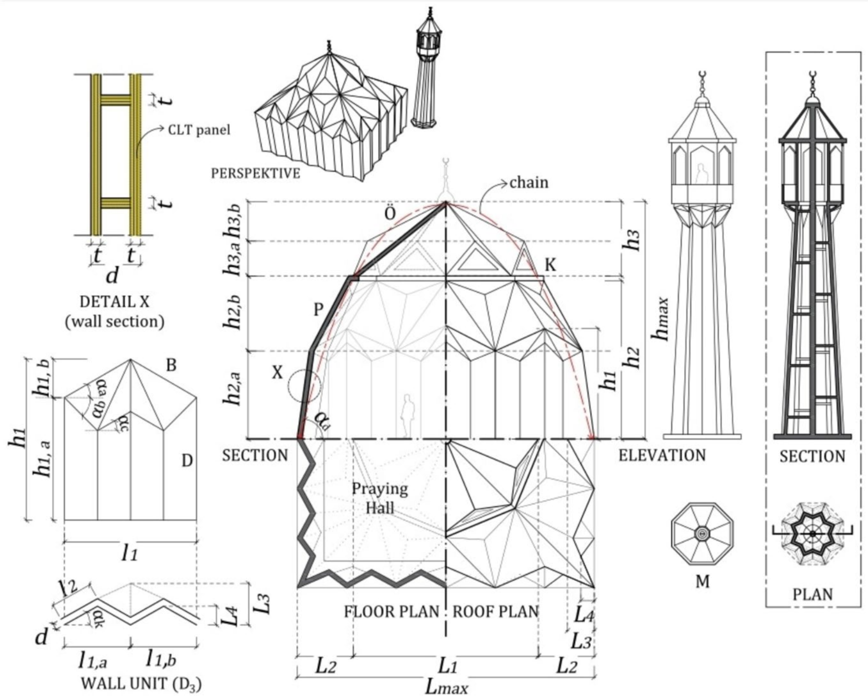


Fig. 3 Dimensional parameters of the Barkat model: This schematic summarizes the static geometric framework based on Turkish triangle geometry within a CLT structure, as developed in the previous studies (Akyürek 2023, 2024), serving as the basis for the current parametric investigation

large-scale structures or smaller plate sizes, and a higher unit count is desired (Fig. 4 (below)).

The main subject of this study, and the most variable part of the model, is the triangular transition zone. This zone closely parallels the dome-to-square, column-capital, and minaret-base intersections found in historical structures. Yet, although the TT exhibits an 800-year formal evolution, historical texts rarely provide explicit descriptions of its geometric construction. Early principles appear in Abu'l-Wafa's tenth-century treatise on folding and cutting polygons, though these are primarily planar pattern techniques. Architectural geometry becomes more legible in an anonymous eleventh–twelfth-century manuscript (Necipoğlu 1995) and, more extensively, in the fifteenth–sixteenth-century Topkapı Scroll (Fig. 5), which presents modular, radially divided plans for ribbed and muqarnas domes. Reconstructions by Notkin (1961), Bulatov (2022), and Özdural (2000) similarly focus on these dome types. While none of these sources explain TT directly, they reveal the planar–polygonal logic fundamental to muqarnas, karbandi, and TT systems. In traditional practice, the third dimension was generated from these modular plans using schematic sectional guidelines, providing the geometric foundation from which the present study develops a parametric interpretation of TT.

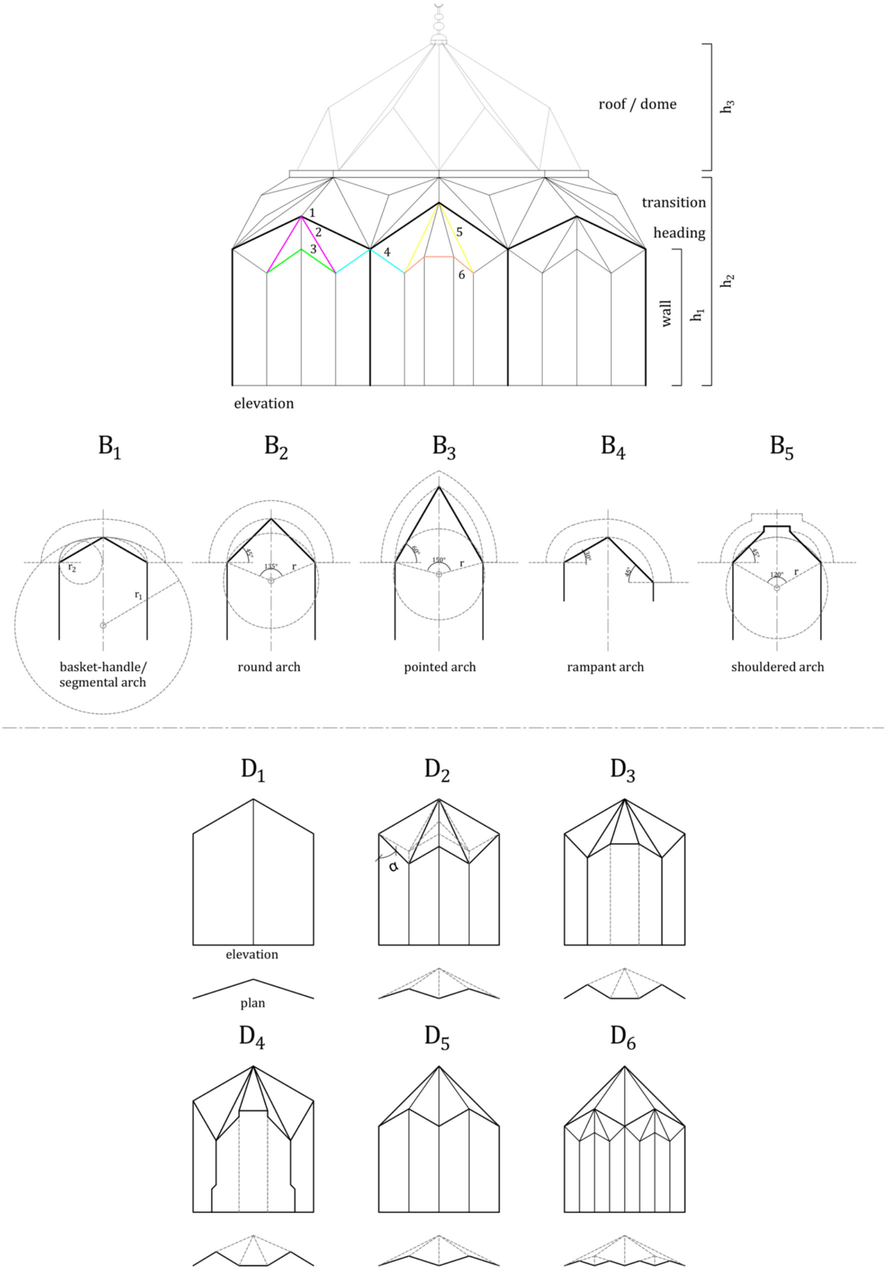


Fig. 4 Topological formation of TT and folded plate modules: Illustrates arch-derived header variations (**B1**–**B5**) and wall unit configurations (**D1**–**D6**) adapted for diverse scales, functions, and rhythmic articulation

Another source of inspiration for this study is Cansever's (2010) doctoral thesis on column capitals, written in 1949. Here, just as in the Topkapi Scroll, there is a radial (virtually; A–H) and symmetrical division in the plan (Fig. 5 (below)). In the plan, the square base represents the impost of the arch, while the central polygonal hoop and circle represent the column. The connection to the column is (approximately) square, polygonal, or stellate. The edges feature single, double, or triple divisions: the second example from the left shows a double division (A_1–B_1) with its symmetry, while the third example shows a triple division (H^1–A_1^1, A_1^1–A_2^1, A_2^1–B^1). This situation is similar to the wall unit repetitions in Barkat.

In line with the relevant references, a design modification method for the transition zone has been developed and is detailed in Table 2, based on Barkat's plan and elevation. The geometry of the transition zone is shaped by the number of wall units (n), the lower (a) and upper (b) points of the wall header, and the number of edges of the dome hoop (k). Triangular planes connect the points constituting the wall header and the hoop (a, b, and k). In this respect, the first two methods correspond to the first and second examples in Table 1. The difference between the third and fourth methods and the first two is that the zigzag starting point is shifted to the adjacent parallel point. If there is a significant size difference between the panels, the larger panels are subdivided into a diamond (fragmentation lines) form, as seen in examples 4, 5, and 6 of Table 1. The number of wall units (n) varies between 1 and 5, while the number of hoop edges (k) varies as 1, 4, 6, 8, 12, etc. In this study, four methods for the transition zone have been derived and explained

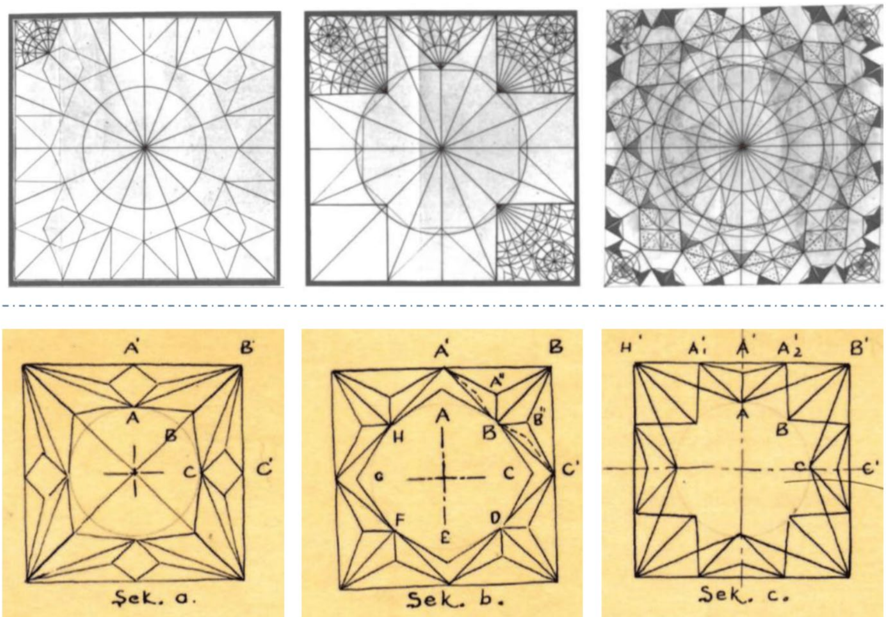
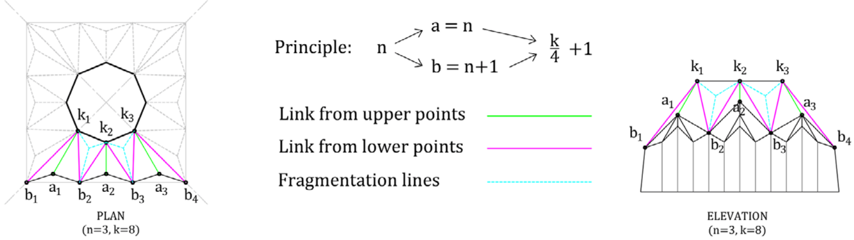
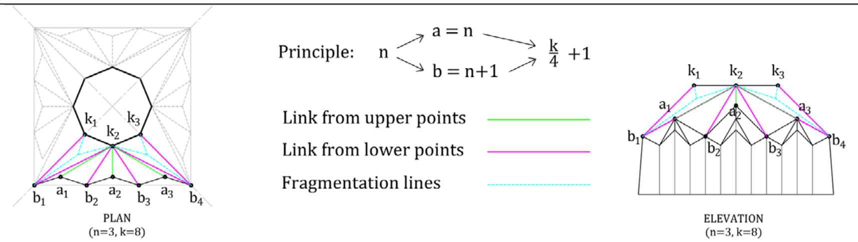
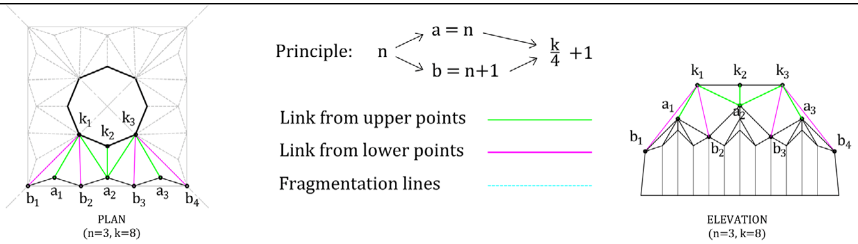
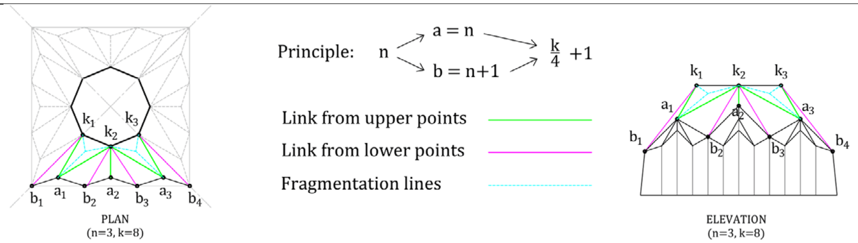


Fig. 5 Plan projection of muqarnas dating back to fifteenth century (Above; Necipoğlu 1995) and plan of certain TT capitals (Below; Cansever 2010)

Table 2 Point-connection rules for the four tested geometric generation methods: the table compares how specific vertex connections determine surface topology and planarity across the tested algorithmic logics

1. Starting from the lower points of the wall header (b), a zigzag is established between the hoop points (k) (pink). Then, the upper points (a) are connected to their corresponding hoop points (green).

2. Starting from the hoop points (k), a zigzag is established between the lower points of the wall header (b) (pink). Then, the upper points (a) are connected to their corresponding hoop points (green).

3. Starting from the upper points of the wall header (a), a zigzag line is drawn between the hoop points (k) (green). Then, the lower points (b) are connected to their corresponding hoop points (pink).

4. Starting from the hoop points (k), a zigzag is established between the upper points of the wall header (a) (green). Then, the lower points (b) are connected to their corresponding hoop points (pink).


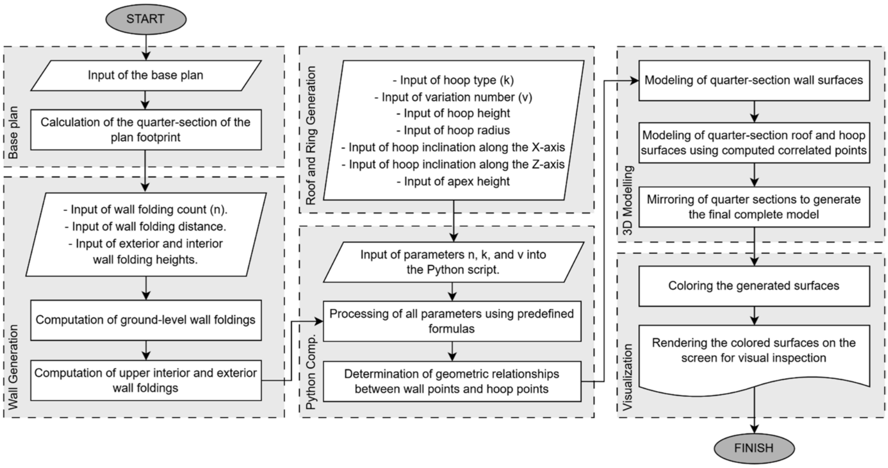


Fig. 6 Algorithm representing the parametric generation logic of Turkish Triangle surfaces

through an example scheme featuring three wall units ($n=3$) and an octagonal hoop ($k=8$). Due to the model's symmetrical (modular) nature, the example is shown only on a single elevation. The variations obtained using this method will be discussed in the findings section.

Parametric Design Definitions

All dimensional and proportional variables (excluding wall thicknesses and the minaret, see Fig. 3) have been defined in the GH according to the four distinct methods discussed in the preceding section. In constructing the parametric design model, the nodal points and linear relationships, much like those in traditional methods (see Fig. 5), have been initially created as "point" and "line" objects on the plan view. Subsequently, these nodal points and relationship lines have been classified within the GH, and their Z-coordinates have been parameterized. The linear wireframe of the desired form was established by assigning heights to relative positions based on these classifications. The final structural volume has been realized by filling the enclosed areas defined by these "line" objects. The geometric generation logic described above is operationalized via a Python script within Grasshopper. The step-by-step algorithmic procedure, which establishes the coordinate-based relationships between the wall folding count (n) and the hoop edge count (k), is detailed in algorithm (Fig. 6).

The parametric design extension and parameters are illustrated in Fig. 7. The parametric extension for the wall and dome geometry (see Fig. 3 and 4) has been successfully formulated without requiring external application plug-ins. However, the TT geometry, used explicitly for the Barkat's transition zone and column capitals, presents infinite variations due to its inherent complexity. Consequently, the relevant parametric modeling has been restricted to the third method outlined in Table 2 and generated using a specialized code developed through GH's Python module.

The wall and dome components have been seamlessly integrated into the system. Although historical structures and the Barkat are based primarily on a square plan, the model's adaptability to rectangular and other plan types has also been tested. For historical adaptations, such as column capitals, the wall and roof components have been removed, and the header offsets have been zeroed. Column capitals have been generated by applying all heights in the reverse (downward) direction.

Findings

In this section, design variations generated by the parametric model are presented based on the specific intervals of hoop edges (k) and wall folding units (n). The analysis focused on even numbers for hoop vertices (k = 4, 6, 8, 12) to maintain the structural symmetry characteristic of traditional geometric analyses and historical heritage. Odd numbers and values exceeding k = 12 have been excluded from the findings matrix, as they result in asymmetric plans or unnecessarily complex forms with diminishing structural returns. Similarly, the number of wall units (n) have been rationalized to a range between 1 and 4 to constrain infinite possibilities into a meaningful and coherent set of variations. The diamond infills (fragmentation lines) of the large triangles, have been omitted to preserve the model's legibility. Additionally, the model's capacity has been tested on diverse boundary shapes -including trapezoidal and polygonal plans- demonstrating its potential for broader architectural applications beyond standard square bases.

Initially, the scenario where the hoop is absent, consisting solely of a single apex point ($k = 1$) have been addressed and mapped onto the plan view (Fig. 8). In this simplest configuration exemplified for the Barkat, all lower (b) and upper (a) points of the wall header converge at this apex, a variation not found in traditional applications. This scenario is included to demonstrate the most minimal variation of Barkat and to test the algorithm's boundary conditions. Subsequently, cases

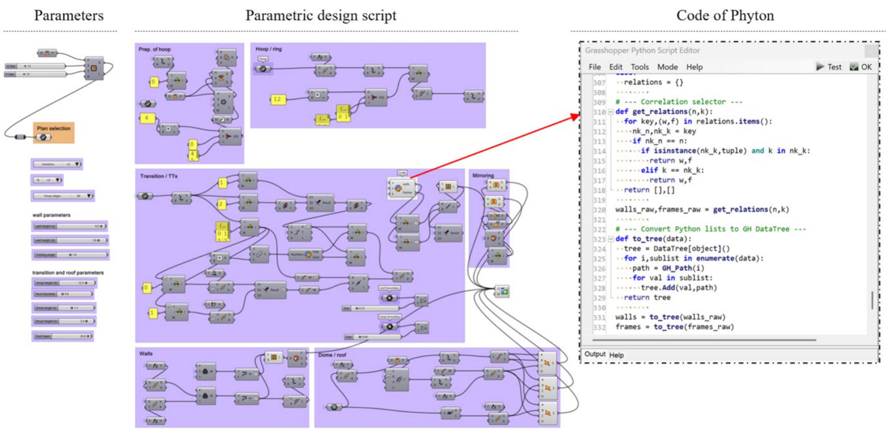


Fig. 7 Extension and parameters of design modification method in the GH

involving hoops with 4, 6, 8, and 12 sides, and walls with 1, 2, 3, and 4 units, have been analyzed according to the four methods in Fig. 9. For every resulting shape, a construction sequence has been generated and annotated beneath the plan, based on the assembly priority of the wall units meeting the hoop. A single sequence suffices since hoop edge counts of $k=4$, $k=8$, and $k=12$ are bi-symmetrical. However, as the $k=6$ case presents two different façade possibilities, the corresponding sequence has been explicitly noted alongside the relevant plan.

In each of the methods examined, all combinations yielded distinct results. When the methods are compared, some overlaps have been observed between the first and the second method, as well as between the third and the fourth method. In the second method, the row $n=1$ and the column $k=4$ yielded the same result as the first method (Fig. 9), which is attributed to the limited scope of the pattern. In the fourth method, the rows $n=1$ and $n=2$ with the column $k=4$ produced results identical to those of the third method. No overlaps have been found in other comparisons, such as when the first method is compared with the third, or the second method with the fourth.

When the number of wall and hoop units increases ($n=4$, $k=12$ or $n=3$, $k=12$), all methods result in triangular panels with disproportionate and unbalanced dimensions. For this reason, the $k=12$ case, which is also rarely seen in traditional applications, is unsuitable for modeling and construction. Combinations of wall units between $n=1$ and $n=3$ with 4, 6, and 8 hoop edges are the most favorable options. Nevertheless, to avoid structural weaknesses and maintain panel sizes close to each other, some larger triangular panels must be reduced using diamond subdivisions (highlighted in gray).

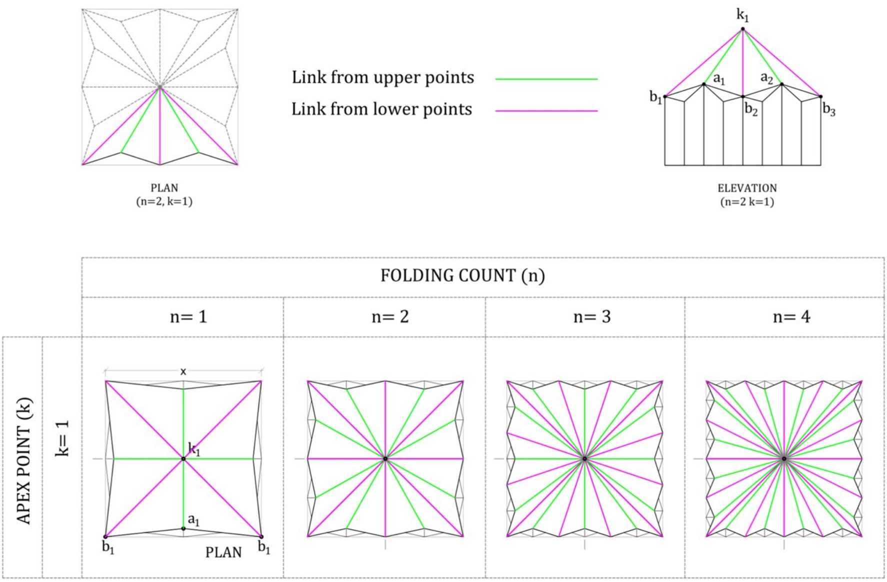


Fig. 8 Geometric adaptation under constrained conditions (Case k = 1): Illustrates topological modifications required when transition zone space is minimized

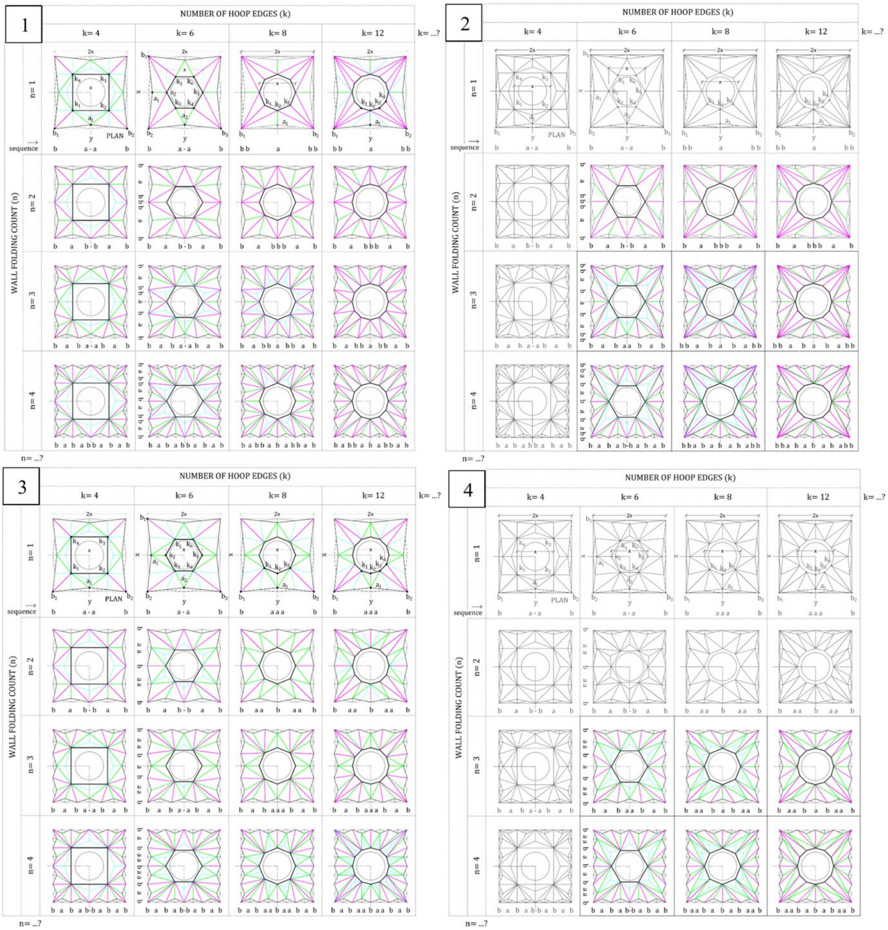


Fig. 9 Comparative analysis of the four algorithmic generation methods: This figure displays the resulting surface topologies derived from different point-connection rules (Methods 1–4)

Discussions

After the consistency of the 2D (plan) analyses has been observed, the relevant relationships have been discussed in the 3D through parametric design. Every example shown in Fig. 9 comprises equivalent corner modules delimited by horizontal and vertical symmetry axes. Consequently, the parametric design modeling has been created by mirroring the developed single corner module. Since the transition zone is formed by the connection between the wall and the dome, the relationship between the wall sequence (b-a-b-a…) and the hoop vertices (k1, k2…) has been structured as a matrix presented in Fig. 10 and coded using GH's Python module. Subsequently, the consistency of the results from the parametric model has been compared with the examples shown in Figs. 9 and 10.

The base of the Barkat, or a column capital, is inherently rectangular or square in shape. Accordingly, a "number slider" has been used in the parametric model to generate this base geometry. The model's operational integrity has been verified by matching the examples in Figs. 9 and 10 one-by-one with their counterparts in the parametric design (Fig. 11a). The adaptability of the model extends beyond these standard typologies, offering a flexibility rarely achievable in traditional masonry. While the base of the Barkat is inherently rectangular or square, the parametric model allows it to respond to irregular urban plots, a challenge Sinan often mastered by skillfully adapting his complexes to the organic pattern of Ottoman cities. Unlike traditional construction, which often dictates strict adherence to regular geometric bases for stability, this computational flexibility allows the Barkat system to fit into non-orthogonal, trapezoidal, or polygonal site boundaries without compromising its typological identity (Fig. 11b). This capability is particularly vital for contemporary urban infill projects where regular plots are scarce. This application makes it possible to generate capitals for polygonal columns or TT zones within various forms, similar to the eighth example in Fig. 1. Furthermore, it offers flexibility that can be leveraged in the design of folded-plate roof constructions, in addition to the analysis of Barkat and traditional structures. This feature is also achieved by dynamically adjusting the relative positions of the corner points of a rectangular base (Fig. 11c). The complete Barkat is formed by integrating the wall and roof units (Fig. 11d). Here, the number of wall units, the wall header, and the dome configurations work in perfect harmony with the transition zone.

Following this typological verification, a critical evaluation of the geometric variations generated by the parameters (n) and (k) reveals significant parallels between the model's structural logic and Sinan's historical preferences. As detailed in Tables 3 and 4, while the model allows for high-frequency variations (e.g., $k = 12$),

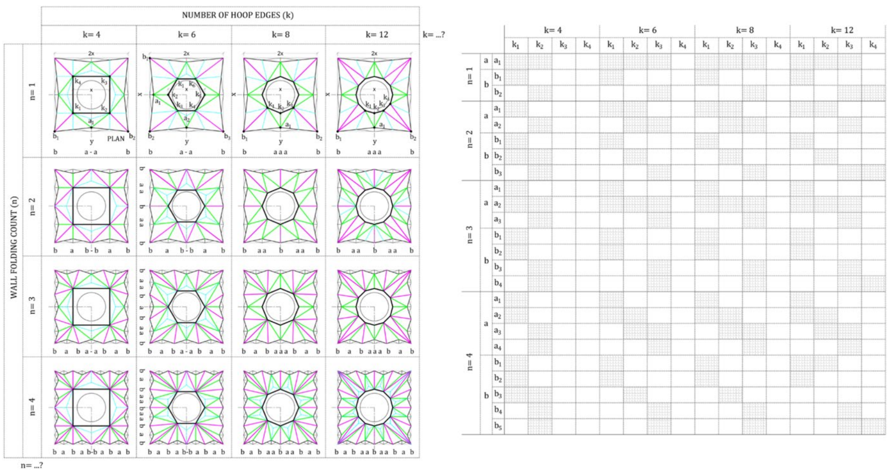


Fig. 10 Planimetric matrix of transition element positioning based on Method 3; Correlates wall folding (n) and hoop edge (k) parameters to demonstrate the systematic positioning (b-a-b…) of transition elements

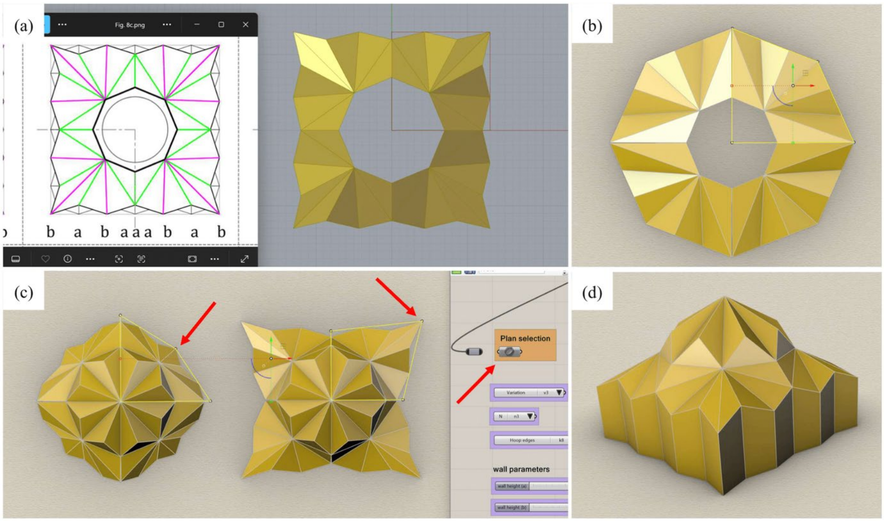
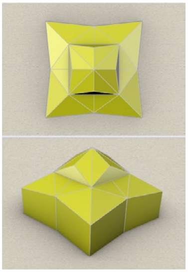
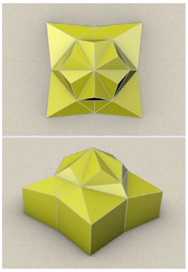
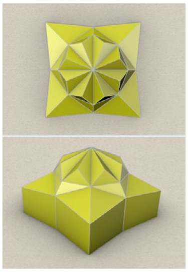
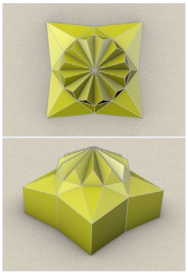
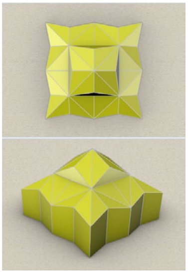
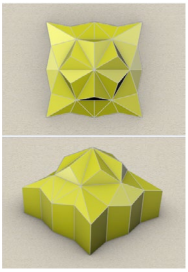
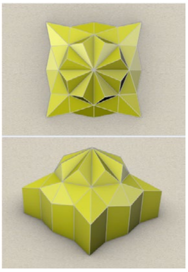
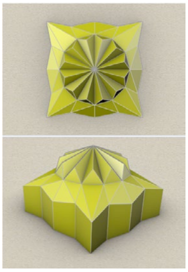
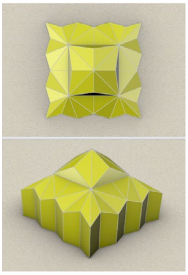
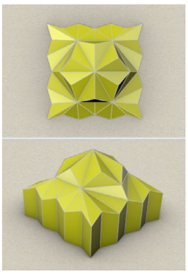
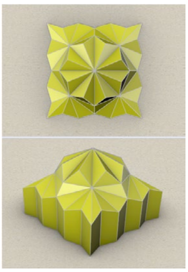
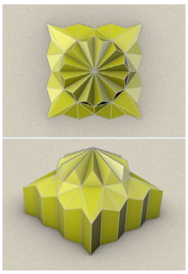
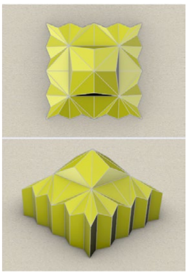
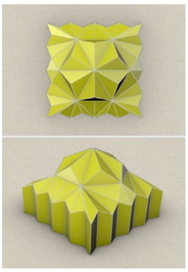
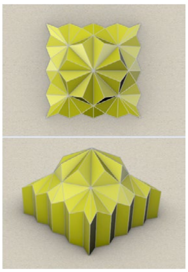
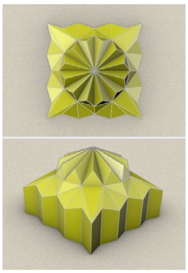


Fig. 11 Geometric adaptability and structural integration: Variations **a**–**c** demonstrate versatility across rectangular and polygonal plans, while **d** illustrates the synthesis the complete assembly

these configurations result in visually chaotic and excessively fragmented surfaces. This finding aligns with Sinan's architectural practice, where 4, 6, or 8-sided baldachin schemes are dominant, while 12-sided forms are rare. From a structural engineering perspective, the depth of the fold is critical; it functions analogously to the web of a beam. In the Barkat model, increasing the folding frequency ($k \geq 12$, $n \geq 4$) flattens this cross-sectional depth, thereby diminishing the system's bending/buckling resistance and transforming an efficient folded-plate structure into a less stable thin shell. Conversely, minimal values ($n = 1$, $k = 1$) produce oversized panels impractical for assembly. Therefore, the optimal range identified ($k = 4$–8) is not merely an aesthetic choice but a constructional necessity, mirroring the rational geometric economy found in Classical Ottoman architecture.

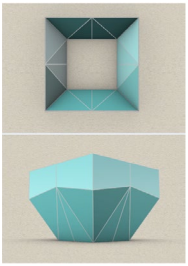
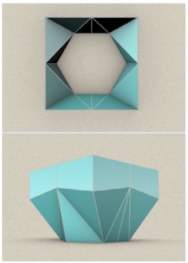
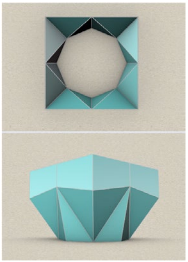
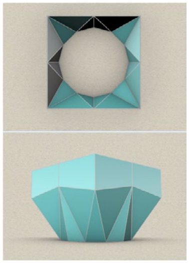
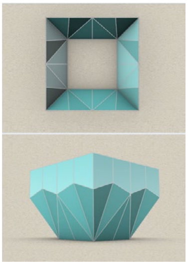
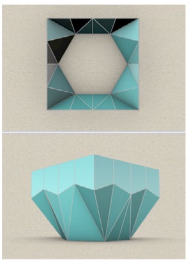
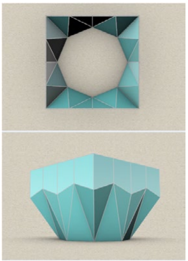
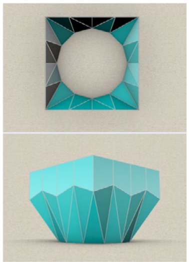
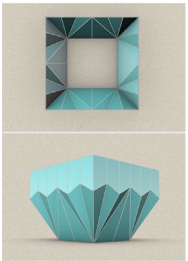
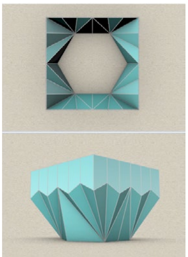
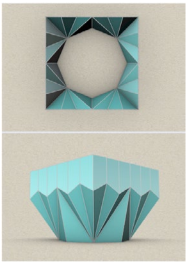
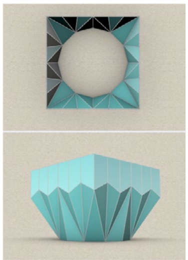
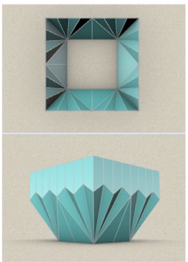
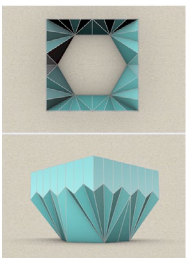
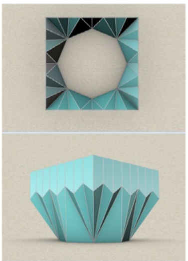
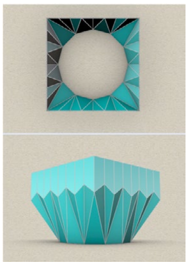
Finally, regarding the Turkish triangle variations, while this study specifically detailed the third generation method, the parametric framework also reveals novel design variations unseen in architectural heritage (Table 4). In historical practice, the wall vertices (*a* and *b*) generally align on a single plane to maintain masonry bond integrity. However, the proposed computational model allows for the misalignment of these vertices or the folding of the wall unit (n) itself, as observed in the Barkat superstructure. This introduces a rich vocabulary of forms where the n unit is not merely a flat surface but a three-dimensional folded element, multiplying the possible derivatives. Consequently, this method serves a dual purpose: it acts as a tool for the geometric reconstruction of historical TTs and, more importantly, provides a generative system that expands the formal vocabulary of the original typology, offering design richness suitable for new architectural contexts.

Table 3 Design variations of the Barkat model according to the third method

a	k=4	k=6	k=8	k=12
n=1				
n=2				
n=3				
n=4				

n: wall folding count, k: number of hoop edges, a: Barkat model, b: TT model

Table 4 Design variations of the TT model according to the third method

b	k=4	k=6	k=8	k=12
n=1				
n=2				
n=3				
n=4				

n: wall folding count, k: number of hoop edges, a: Barkat model, b: TT model

Conclusion

The primary contribution of this research is the establishment of a novel computational methodology that bridges the gap between historical documentation and contemporary fabrication. Unlike traditional architectural history studies that primarily focus on the cataloging and typological classification of Turkish triangles (e.g., Batur 1986; Şimşek 2010), this study moves beyond descriptive analysis to generative reconstruction. By converting the static geometric rules of the Barkat model, a transformation of the spatial and proportional design in Sinan's mosques, into a dynamic parametric algorithm, the research demonstrates that these historical forms are rational geometric systems capable of systematic reinterpretation for modern production. In contrast with the author's earlier topological model (Akyürek 2024), which remained a static geometric framework, this study provides a fully generative system whose behavior can be explicitly controlled through the parameters n and k.

Furthermore, this study differentiates itself from existing computational attempts in the field. While previous parametric studies (e.g., Çağlar et al. 2021) have typically focused on singular case studies or surface-level pattern generation, this research proposes a comprehensive typological grammar, the Barkat model. This framework does not just mimic a shape; it establishes a structural logic compatible with CLT technology. The algorithm developed here proves that the complex transition from a square base to a circular dome can be rationalized into a production-ready folded-plate system, offering a versatile tool adaptable to non-orthogonal and irregular site boundaries, a capability previously absent in both restoration practice and contemporary mosque design literature.

A significant empirical finding of this investigation is the identification of geometric thresholds necessary for both structural balance and aesthetic coherence. The algorithmic analysis of wall folding (n) and hoop edge (k) parameters demonstrates that valid typologies are confined to specific intervals, namely $n=1$–3 and $k=4$–8. Within this range, the generated triangular surfaces maintain optimal aspect ratios, ensuring load-bearing efficiency and feasibility for CLT panel fabrication. Conversely, exceeding these thresholds (e.g., $k \geq 12$) results in excessive fragmentation, producing needle-like surfaces that are structurally unstable and inefficient to manufacture. This computational limitation remarkably parallels the material constraints observed in historical masonry. Thus, the result mathematically validates Sinan's intuitive use of proportion, demonstrating that the Classical Ottoman sense of 'geometric harmony' is not merely a stylistic preference, but a quantifiable logic grounded in structural rationalism.

Consequently, while this study validates the geometric definition and digital fabrication feasibility of the model, certain performance aspects remain open for further investigation. The applicability of the generated forms for CNC fabrication has been initially verified within the scope of a scientific research project; however, full-scale structural behavior analyses under various load scenarios are ongoing. Future research will focus on these structural simulations, as well as building physics analyses, specifically daylight optimization and thermal comfort within the multi-faceted folded plate geometry. Additionally, the aesthetic and practical detailing of cladding materials for this parametric shell constitutes a distinct avenue for future architectural development.

Funding Open access funding provided by the Scientific and Technological Research Council of Türkiye (TÜBİTAK).

Data Availability Drawings and models of the paper can be shared by the corresponding author upon request.

References

Aktuğ Kolay, İ. 2017. *Batı Anadolu 14. Yüzyıl Beylikler Mimarisinde Yapım Teknikleri*. Ankara: Atatürk Kültür Merkezi Yayınları.

Akyürek, M. E. 2023. *Mescit mimarisinin kavramsal çerçevesinde Sinan'ın camilerini günümüze bir uyarlama modeli*. İstanbul Sabahattin Zaim Universitesi, Lisansüstü Eğitim Enstitüsü.

Akyürek, M. E. and Kahraman, G. 2021. Sinan'ın Camilerinde Yapı Mühendisliği Sanatı. *Sanat Tarihi Dergisi*, 30 (1), 255–283. https://doi.org/10.29135/std.817575

Akyürek, M. E. 2024. A Topological and Technological Transformation Model of Mimar Sinan's Mosques to the Present. *Journal of Islamic Architecture*, 8(1), 91–105. https://doi.org/10.18860/jia.v8i1.23694

Arseven, C. E. 2017. *Osmanlı Dönemi Mimarlık Sözlüğü: Istılâhât-ı Mi'mâriyye*. İstanbul: Kaknüs Yayınları.

Batur, A. 1980. *Osmanlı Camilerinde Örtü ve Geçiş Öğeleri: Sorunlar ve Biçimler*. İstanbul Teknik Üniversitesi Mimarlık Fakültesi.

Batur, A. 1986. Osmanlı Camilerinde Geçiş Öğelerinin Evrimi. *II. Uluslararası Türk ve İslam Bilim ve Teknoloji Tarihi Kongresi: Mimar Sinan*, (V2, p. 207–216), İstanbul: İTÜ İnşaat Fakültesi Matbaası.

Bulatov, M. S. 2022. *Geometric Harmonization in the Architecture of Central Asia (9th–15th Centuries): Historical-Theoretical Research*. Translated by L. Sattarova. İstanbul, Türkiye: IRCICA.

Çağlar, B., Bor, H. O. and Vural, S. 2021. Türk Üçgeninin Parametrik Modellenmesi ve Robotik Üretimi Üzerine Bir Yöntem. *Mimarlıkta Sayısal Tasarım XV. Ulusal Sempozyumu*, İTÜ, İstanbul.

Cansever, T. 1949. *Türk Sütun Başlıkları*. Istanbul University.

Cansever, T. 2010. *Sonsuz Mekanın Peşinde: Selçuk ve Osmanlı Sanatında Sütun Başlıkları*. İstanbul: Klasik Yayınları.

Komisyon. 2015. *Osmanlı Mimarisi: Usûl-i Mi'mârî-i Osmânî*. İstanbul: Çamlıca Yayınları.

Kula Say, S. 2011. *Kubbeye Doğru: Erken Dönem Osmanlı Hamamlarında Eğrisel Örtüye Geçiş Sistemleri*. İstanbul: Tarihçi Kitabevi.

Morita, M 2017. Historical Development on Transitional Zone of Dome Employing Triangles in Islamic Mausoleums in Medieval Anatolia. *Journal of Architectural and Planning*, 82 (741), 3001-3011. https://doi.org/10.3130/aija.82.3001

Necipoğlu, G. 1995. *Topkapi Scroll: Geometry and Ornament in Islamic Architecture*. Edited by J. Bloomfield, T. F. Reese, and S. Settis. Santa Monica, CA: Getty Center Publications.

Necipoğlu, G. 2017. "Ornamental Geometries: A Persian Compendium at the Intersection of the Visual Arts and Mathematical Sciences." In *The Arts of Ornamental Geometry*, edited by G. Necipoğlu. Boston, MA: Brill.

Nişanyan Sözlük. Bark. Çağdaş Türkçenin Etimolojisinde. September 2, 2025, https://www.nisanyansozluk.com/kelime/bark

Notkin, I. I. 1961. *Buxarskaya rez'ba po ganchu v rabotax Usto Shirina Muradova*. Tashkent, USSR: Gosudarstvennoe izdatel'stvo xudojestvennoy literaturi UzSSR.

Okçuoğlu, T. 1995. *Anadolu Selçuklu Mescitlerinde Kubbeye Geçiş Alanının Değerlendirilmesi*. İstanbul Üniversitesi, Sosyal Bilimler Enstitüsü.

Özcan, A. 2008. 14-15. *Yüzyıl Bursa Cami ve Mescitlerinde Kubbeye Geçiş Elemanları*. Erciyes Üniversitesi, Sosyal Bilimler Enstitüsü.

Özdural, A. 2000. Mathematics and Arts: Connections between Theory and Practice in the Medieval Islamic World. *Historia Mathematica* 27: 171–201. https://doi.org/10.1006/hmat.1999.2274

Şimşek, H. 2010. *Erken Osmanlı Mimarisinde Kubbeye Geçiş Sistemlerinden Üçgenler Kuşağı*. Yüzüncü Yıl Üniversitesi, Sosyal Bilimler Enstitüsü.

Turan, Ş. N. 2018. *Türk Mimarisinde Kullanılan Kubbeye Geçiş Elemanları; 13. yy. Anadolu Selçuklu Dönemi Konya Mahalle Mescitleri Örneği*. Necmettin Erbakan Üniversitesi, Fen Bilimleri Enstitüsü.

Uluengin, F., Uluengin, B. and Uluengin, M. B. 2016. *Osmanlı Anıt Mimarisinde Klasik Yapı Detayları*. İstanbul: Yem Yayın.

Useev, N. 2016. Manas Destanı'nda "Bark" Kelimesi ve Eski Türk Mezar Geleneği. *Turkish World Journal of Language and Literature,* (42), 173-191.

Erdem Köymen graduated from Trakya University's Department of Architecture in 2005 and completed his Master's degree in 2008, focusing on "Architecture in 3D Animated Films." He earned his Ph.D. from Yıldız Technical University in 2014 with a dissertation on augmented reality-assisted real-time 3D sketch modeling. Throughout his career, he has contributed to architectural design, 3D animation, game development, and music composition. He has also worked as an architect, game developer, graphic designer, art director, and scene designer. Currently, he teaches at Istanbul Zaim University's Architecture, Interior Architecture, Industrial Engineering and Software Engineering Departments and continues research in architectural design, modeling, animation, and digital arts.

Muhammed Emin Akyürek has been serving as an Assistant Professor in the Department of Architecture at Istanbul Sabahattin ZaimUniversity since 2023. His research interests include the structural systems of Mimar Sinan's mosques and structural form-findingmore broadly. His current work focuses on the rational adaptation of traditional structural forms to contemporary constructiontechniques and materials.