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Effect of demand uncertainty on omnichannel distribution network design strategies

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Abstract

To investigate the effect of uncertain demand on various omnichannel distribution network strategies and analyze the impacts of various online order fulfillment policies, a two-staged network optimization model is built. The two stages are a deterministic model to define the optimal distribution network and a simulation model to determine the optimal network flow to maximize the total profit for a given distribution network and online order fulfillment policy. This study shows the importance of taking demand uncertainty, requested delivery time, and customer service level (CSL) into account in designing an omnichannel distribution network. The results of the simulation model indicate that shipments from warehouse (SFW) and hybrid strategies more successfully cope with variation because there is a greater profit decline in shipments from store (SFS) strategies. The results also show that applying a dynamic assignment policy plays an important role in handling variation in demand. The study shows that, as demand uncertainty and CSL increase, having products available in central repositories becomes even more important to cope with demand variation. The study also indicates SFS strategies that seem more profitable in the assumed-to-be deterministic model might be misleading because SFS strategies become less profitable (compared to SFW strategies) when demand uncertainty is taken into account.

Keywords: omnichannel distribution; demand uncertainty; distribution network; distribution network optimization

1. Introduction

With the advances in mobile Internet, social media, and changing consumer behaviors, people increasingly rely on their smartphones for shopping (Verhoef et al., 2015). They want to order whenever it is convenient for them, and they want the overall shopping experience to be seamless (PWC, 2017). This has resulted in a new channel approach called the omnichannel that requires the integration of all customer channels. Omnichannel retailing is gaining more attention among practitioners

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and academicians (Galipoglu et al., 2018). However, research on the omnichannel is often undertaken from a sales and marketing perspective (Saghiri et al., 2018). There is a need for more papers investigating the optimal design of the physical distribution and operations side of omnichannel retailing (Galipoglu et al., 2018) because, in an omnichannel strategy, marketing and logistics issues are closely related (Melacini et al., 2018).

One of the main questions to be answered when designing the omnichannel distribution network is where inventory should be stored and distributed to online customers (Hubner et al., 2016). The design of an omnichannel distribution network plays a critical role in the profitability and competitiveness of omnichannel retailers. For example, Govindarajan et al. (2021) showed that *Target* could substantially increase its profit by pooling the inventory held at stores with the inventory held at centralized inventory points, whereas giant retailers such as *Walmart*, *Carrefour*, and *Amazon* have redesigned their omnichannel distribution networks to get closer to the customers without hurting the efficiency of the distribution network (Mohamed et al., 2020).

A review of the relevant literature shows that the number of studies analyzing the performance of various omnichannel distribution network designs is rapidly increasing. However, a majority of those studies avoid modeling demand uncertainty because of the complexity of stochastic modeling, even though most retailers state they need to deal with demand-related uncertainties when designing their omnichannel distribution networks (Hubner et al., 2022). Moreover, the number of stochastic omnichannel distribution network design studies is also increasing (e.g., Modak and Kelle, 2019; Arslan et al., 2021; Bayram and Cesaret, 2021; Govindarajan et al., 2021). In those stochastic studies, the omnichannel distribution network is optimized and analyzed with the definition of stochastic demand without analyzing the impact of uncertainty in demand. In the present study, the performances of the various distribution network design strategies are analyzed in a two-stage procedure to explore the impact of demand uncertainty and understand how and when including demand uncertainty is more critical. To conduct the analysis, the following two-stage network optimization model was developed where strategic level decisions are made before the realization of uncertain demand. Two stages are: (a) a deterministic optimization model, which determines the optimal distribution strategy in terms of the ideal number, size, and capacity of the warehouses in a given channel design strategy, and (b) a simulation model, which considers the variation in demand and evaluates various order fulfillment strategies to maximize the total profit for a given scenario. In the study, the effects of the dynamic assignment policies under various omnichannel distribution strategies are also investigated as more studies are needed to understand when and how cost reduction due to dynamic assignment of online orders is maximized (Melacini et al., 2018).

This study aims to address the following research questions:

1. Given the omnichannel distribution strategy (i.e., shipments from warehouse [SFW], shipments from store [SFS], or hybrid strategy), to what degree is total profit affected when demand uncertainty is considered in a seasonal single-product profit maximization model?
2. What is the impact of ignoring demand uncertainty in the selection of omnichannel distribution strategy? Specifically, under which market conditions, deterministic models, ignoring the demand uncertainty, may generate misleading results?
3. How successful are various omnichannel distribution strategies in coping with the increased level of demand uncertainty?

4. When is it more critical to include demand uncertainty considering the varying levels of demand uncertainty, online demand ratio, desired product availability, and requested response time?
5. What is the impact of the requested response time on the profitability of the various omnichannel distribution strategies?
6. What is the additional profit the firms may expect when the market and fulfillment center assignments are removed, which mathematically means that the flows between shipment points and online markets are split, under different omnichannel distribution strategies?
7. Finally, what is the impact of the dynamic assignment policies under different distribution strategies with varying levels of demand uncertainty, online demand ratio, customer service level (CSL), and requested response time?

The main results of the study can be summarized as follows: First, the study shows that demand uncertainty should be taken into account when designing the omnichannel distribution network even when the level of variation in demand is low. SFS strategies seeming more profitable in the deterministic model might be misleading because SFS strategies become less profitable (compared to SFW strategies) when demand uncertainty is taken into account. Second, the results indicate that SFW and hybrid strategies more successfully cope with variation because there is a greater profit decline in SFS strategies. Third, the study also shows the importance of applying a dynamic assignment policy in handling variation in demand. Fourth, the study shows that having products available in central repositories (as in SFW and partly in hybrid strategy) becomes even more important to cope with demand variation as demand uncertainty and the desired CSL increase. The study's results also indicate that the requested response time for online orders is a critical factor to consider when designing an omnichannel distribution network.

Section 2 of this study provides a general overview of the current distribution-oriented omnichannel literature and highlights the contributions of the current work. Section 3 consists of a detailed definition of the proposed model using mathematical notation and formulations. Section 4 includes the results of the numerical analysis, and Section 5 provides conclusions, managerial insights, and further research suggestions.

2. Literature review

Many pioneering studies have analyzed the logistics associated with e-commerce (e.g., Koster, 2002; Swaminathan and Tayur, 2003). Since the publication of the pioneering works, the number of studies analyzing the distribution side of the omnichannel has substantially increased. The vast majority of these studies fall into the following three categories: (a) those that redesign the distribution network or evaluate the optimal choice of channel design to serve online customers, (b) those that focus on inventory optimization and assortment planning across multiple channels, and (c) those that focus on the dynamic assignment of online orders to warehouses or stores.

Regarding the first category, many papers have addressed the use of either store (SFS) or warehouses (SFW) for fulfilling online demand. Alptekinoglu and Tang (2005) developed a multichannel distribution system subject to stochastic demand to analyze the following two specific applications: using a store (SFS) or a central depot (SFW) to satisfy direct channel demand. Even though the authors used their model to investigate how the results change as the demand correlation

coefficient among sales points changes, they never analyzed the impact of the changing variation of demand. They also point out that their model may be adjusted for the hybrid fulfillment strategy; however, they do not include that in their study. Geng and Mallik (2007) developed a two-level game-theoretic multichannel distribution system to study inventory competition and how manufacturers allocate inventory to competing channels. Liao et al. (2017) proposed a multi-objective approach to optimize transportation-related decisions and the locations of warehouses and inventory. The model included the following three objectives: minimizing location- and inventory-related costs, minimizing transportation costs, and maximizing the satisfaction of online customers. Bendoly et al. (2007) analyzed a two-echelon, fixed-period, order-up-to inventory system to determine when and if decentralizing online inventory is ideal. Hovelaque et al. (2007) also studied the inventory management and ordering policies in a multichannel environment and compared the results of centralized (SFS), decentralized (SFW), and drop-shipping online order fulfillment models using the traditional newsvendor order policy model. Bretthauer et al. (2010) also assessed where and how much inventory should be held in an omnichannel market by optimizing a predetermined subset of online fulfillment centers as the percentage of online demand varies. In a recent study, Arslan et al. (2021) built a two-staged stochastic mixed integer program to investigate the optimality of the following three omnichannel distribution networks: SFW, SFS, and ship from urban fulfillment platforms. Even though the study included demand uncertainty in the optimization model, the study never investigated the impact of demand uncertainty on the design of distribution networks. Millstein and Campbell (2018) optimized the omnichannel distribution system for a sporting goods retailer to determine the optimal number and location of omnichannel distribution centers. Millstein et al. (2022) also studied the performance of various channel design strategies (i.e., SFS, SFW, and hybrid strategy) with varying cost parameters. The literature review on the use of either stores or warehouses to fulfill online demand reveals that the vast majority of the papers that investigated the impact of the different distribution strategies assumed the locations were fixed and ignored the effects of reconfiguring the network in a given distribution strategy. However, redesigning the supply chain (SC) network, including the facility locations, market assignments, and network traffic flows, is a key decision in the chain (Agatz et al., 2008). There are only three studies that include the location of the distribution center as a decision variable (Liao et al., 2017; Millstein and Campbell, 2018; Millstein et al., 2022). However, those studies did not analyze the effect of demand uncertainty. In the present study, the omnichannel distribution network optimization model developed by Millstein et al. (2022) is extended to include varying levels of demand uncertainty. Their model included strategic level facility-based decisions and operational level traffic flow decisions in redesigning the SC network and considered various omnichannel distribution strategies, including SFS, SFW, and hybrid.

Regarding the second category, many papers have addressed inventory optimization and assortment planning for the online channel. Liu et al. (2010) built a capacitated facility location model to seek the optimal tradeoff level between inventory holding cost savings and transportation costs as demand variance changes. Li et al. (2015) built a multi-period stochastic dynamic programming model to investigate how demand dependency on inventory levels affects the optimal inventory levels of dual channels. Xu et al. (2017) incorporated advance demand information in a multichannel mixed integer linear programming (MILP) inventory replenishment planning problem. Alawneh and Zhang (2018) also built a multi-item inventory model that considered warehouse capacity constraints, demand, and lead time uncertainty to optimize the inventory for an omnichannel

warehouse. Govindarajan et al. (2021) developed a two-stage model to jointly optimize inventory levels and fulfillment decisions. In their study, the authors investigated the impact of demand spillovers across the network. The study showed the value of centralized inventory planning in addition to the inventory held at physical stores (i.e., a hybrid strategy) and explored when contribution by centralized inventory points was highest. Fan et al. (2019) constructed dual-channel inventory optimization models to analyze the impact of the following two special cases: inventory competition brought on by customer shift and inventory cooperation brought on by inventory transshipment.

Several studies regarding distribution-oriented omnichannels address the assignment of online orders to the retailer's given warehouse or store location. Bendoly (2004), in a two-echelon SC network serving in-store and online customers, investigated the impacts of increased substitute availability of neighboring stores for online shipments. Mahar et al. (2009) developed two dynamic assignment policies to examine how those policies could help companies that fulfill both in-store and online demand. In another dynamic assignment study, Mahar and Wright (2009) developed a quasi-dynamic allocation policy that assigns postponed sales to the optimal fulfillment locations. Bhatnagar and Syam (2014) developed a profit maximization model to determine the optimal allocation of products with different characteristics (e.g., low and high carrying costs) among central depots or stores in a multichannel SC. Jasin and Sinha (2015) optimized fulfillment decisions to minimize expected total fulfillment cost over the entire horizon by considering future demand distribution and inventory positions. Lei et al. (2018) optimized the assignment of online orders to fulfillment centers to maximize the expected total profit, which is calculated as the difference between the expected profit and shipping cost in a multiple-period setting. In a recent study, Bayram and Cesaret (2021) investigated the implementation of an SFS model to fulfill online orders and proposed a dynamic order fulfillment model in which the online orders are fulfilled either from stores or online fulfillment centers.

In other studies, researchers have analyzed only one critical aspect of omnichannel distribution models; they never investigate the distribution network strategy as a whole. Cao et al. (2016), Gao and Su (2017), and Gallino and Moreno (2014) evaluated the potential benefits of adding a buy-online-and-pickup-at-store (BOPS) to the existing store channel strategies. Peng et al. (2022) analyzed the impact of revenue sharing in the application of BOPS under various market conditions. Yang and Zhang (2020) investigated the ship-to-store policy with and without the quick replenishment policy in an omnichannel newsvendor model to understand the interaction between the ship-to-store policy and the quick replenishment policy. Xu et al. (2021) also developed an omnichannel newsvendor model to analyze the effect of a store pickup policy for online sales on pricing and ordering decisions during online shopping carnivals. Li et al. (2015) analyzed retail strategies for a given assortment, logistics costs, and consumer behaviors.

Aside from omnichannel distribution, in the literature on traditional SC network optimization, there are many papers (e.g., Azaron et al., 2008; Zokaee et al., 2017) that have studied the impacts of uncertainty on the SC network. Daskin et al. (2002) developed a distribution center location model for a situation in which retailers' demands have a normal distribution with a known mean and variance. That study has been used as a foundation for many other studies in which the retailers' demands have a normal distribution with a known mean and variance (Govindan et al., 2017). Azaron et al. (2008) developed a two-stage multi-objective stochastic model approach for SC design under uncertainty. They considered the binary variables, which characterize the network configuration as first-stage variables. Decision variables regarding the flow of materials are considered second-stage

variables that correspond to the decision made after the uncertain parameters have been revealed. Zokaee et al. (2017) developed a network optimization model to design a real-life bread production and distribution SC facing uncertainty in demand. The authors first optimized the deterministic model to determine the strategic location and tactical allocation decisions. Then, they extended the model to incorporate uncertainty to generate some managerial insights and practical implications, including the impact of demand uncertainty. Kiliç and Tuzkaya (2015) developed a two-stage stochastic mixed integer programming model to handle demand uncertainty for the design of the distribution network of a home appliance company. Biçe and Batun (2020) built a profit maximization closed-loop SC network design model. They formulated the problem as a two-stage stochastic MILP and investigated the impact of varying levels of uncertainty in demand. Azaron et al. (2021) developed a multi-objective two-stage programming model to maximize the expected net present value and minimize the expected travel time in SCs under demand uncertainty. As the literature review reveals, demand uncertainty is a well-studied subject in single-channel SC networks. Also, a two-staged optimization approach is often used that determines strategic-level decisions in the first stage and then optimizes operational- and tactical-level decisions after the realization of uncertain parameters.

The number of stochastic omnichannel distribution network design studies is rapidly increasing, as detailed in the literature review. However, in those studies, the omnichannel distribution network is optimized using the definition of stochastic demand; the impact of uncertainty in demand has not been deliberately investigated. There is only one study (Alptekinoglu and Tang, 2005) that has investigated the effects of varying levels of demand correlation among sales locations. However, the model presented in that study leaves out several essential components of distribution modeling, such as location-based decision variables and inventory held at warehouses. Moreover, the study did not examine the hybrid distribution channel strategy, and it ignored varying levels of demand uncertainty. Considering the effect of demand uncertainty is a well-studied subject in the traditional SC network optimization literature, the impact of varying levels of demand uncertainty on omnichannel distribution network strategies still needs to be investigated. As such, this study contributes to the existing literature in several ways. First, the two-staged approach investigates how the total expected profit of the SC changes when demand uncertainty is not taken into account. The effects of varying levels of demand uncertainty on various distribution network strategies are investigated by comparing expected and realized profits. Second, the model analyzes the impact of the market and fulfillment center assignment constraints and the impact of dynamic assignment strategies under different distribution network settings. The study also presents one of the few comprehensive omnichannel distribution network optimization models that optimize facility locations to investigate the optimal distribution network strategy.

3. Model definition

3.1. Problem description and modeling assumptions

It is a strategic-level decision for companies to design their omnichannel distribution network. Omnichannel distribution network design strategies are classified according to how online demand is fulfilled (i.e., where the inventory is coming from) and include the following: SFW, in which

warehouses serve retail stores (RSs) both for retail demand and to fulfill online orders; SFS, in which both retail and online demand is fulfilled by RSs and no inventory is held at central warehouses (Bendoly, 2004; Hubner et al., 2016); and a hybrid design, which allows warehouses to both serve RSs and fulfill online orders from either central warehouses or RSs (Govindarajan et al., 2021; Millstein et al., 2022). Many papers have investigated the optimality of those strategies as reviewed in the literature review section. It should be noted that dedicated warehouses for online orders are not considered in the study because integrated warehouses serving both online and offline orders have become the most common warehouse structure for omnichannel distribution networks (Alawneh and Zhang, 2018).

A company needs to design its omnichannel distribution network. When companies design their distribution network, it is very often that they ignore demand uncertainty. The company, in the first stage, ignores demand uncertainty and designs its distribution network for three different channel design strategies. Then, the actual demand on the market is revealed, and the actual total profit is realized by the channel design selection of the company. By doing so, how the profitability of the various channel design strategies change is analyzed when the different levels of demand variation are imposed. Also, the impact of ignoring the demand uncertainty in designing the distribution network is explored under various channel design strategies, with different levels of demand uncertainty and various ratios of online demand.

The model optimizes the number and locations of warehouses by considering a set of candidate warehouse locations, each with a set of candidate warehouse capacities and corresponding fixed and variable costs. Input data for our model include a set of markets. The total demand in a market is apportioned to either online or retail demand based on a changing ratio to allow for experimental control of the online demand as a ratio of total demand in the markets.

The performance measure of the model is total profit. The outputs of the models include the flows of products. The flow of products defines the assignments of warehouses to stores and online markets. The other outputs of the model are the total profit, number, locations, and capacity of the warehouses.

The following assumptions are imposed in the study:

1. In all three distribution network design strategies, RSs are resupplied from a set of warehouses (Fig. 1). Because the products that fulfill retail demand are shipped directly to and stored at RSs, no inventory is held to fulfill retail demand at central warehouses (Bendoly, 2004).

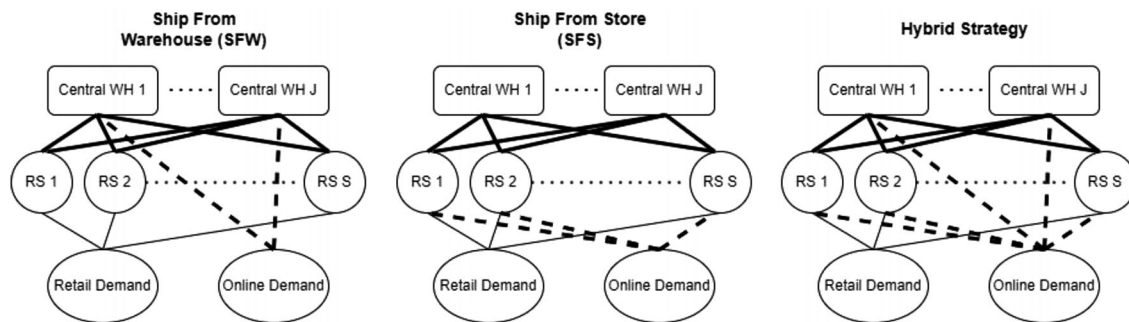


Fig. 1. Product flows for different channel design strategies.

2. A three-echelon (i.e., distribution center, RS, and customers) SC network with multiple distribution centers, a single term, and a single product model is defined.
3. A seasonal product with a long lead time is considered. Because of the long lead times, the inventory decisions are only made once at the beginning of the selling season (Hovelaque et al., 2007; Govindarajan et al., 2021; Zhong et al., 2023).
4. The following two types of independent demand are defined for each customer zone (market): retail demand for retail markets and online demand for online markets (Alptekinoğlu and Tang, 2005; Hovelaque et al., 2007). The retail and online channel demands are both assumed to be normally distributed (Alptekinoğlu and Tang, 2005; Bendoly et al., 2007) and independent from each other.
5. Store demand is fulfilled by the firm's given set of RSs. Online demand is fulfilled from either warehouses or stores, depending on the channel design.
6. In SFW strategy, each online market can be served by a single warehouse (WH), while each online market can be served by a single RS in SFS strategy. However, there is no customer and fulfillment center (either WH or RS) assignment in hybrid strategy.
7. The store network locations are fixed and capacitated. Store inventory is replenished from the warehouse inventory at the beginning of the selling season.
8. All the inventory items shipped to fulfill online demand from a warehouse incur warehouse inventory holding costs, and all inventory items sold by the store to fulfill the online or retail demand incur store inventory holding costs. As store inventory that is used to fulfill retail demand is shipped from the warehouse at the beginning of the season, no inventory holding cost incurs at the warehouse for the retail demand.
9. All online orders (items) fulfilled by the warehouse incur a warehouse handling cost, and all online orders (items) fulfilled by the store incur a store handling cost. Online order handling cost is defined as the labor cost to pick and pack online orders for shipment.
10. Online order handling costs are higher at stores than at warehouses due to the inherent inefficiencies in a store. The handling cost of fulfilling retail orders is ignored in the study because it is the same in any distribution channel design strategy.
11. All fulfillment points (WHs and/or RSs depending on the strategy) keep safety inventory to ensure desired CSL. The level of safety inventory is based on the desired CSL and the level of demand variation.
12. To take delivery time into account, the requested response time for each demand point is defined. If the online orders are not fulfilled within the requested response time, then the demand is lost (Arslan et al., 2021). Response time (in days) increases as an integer value with every 500 miles (Millstein et al., 2022). That is, if the requested response time is one day, online demand only within a radius of 500 miles from the fulfillment point is fulfilled. In the same way, if the requested response time is two days, online demand only within a radius of 1000 miles from the fulfillment point is fulfilled.

3.2. Proposed methodology

Figure 2 summarizes the methodology of the analysis implemented in this article. A two-staged approach (Phase I and Phase II) is adopted from Azaron et al. (2008), Arslan et al. (2021), and Lim

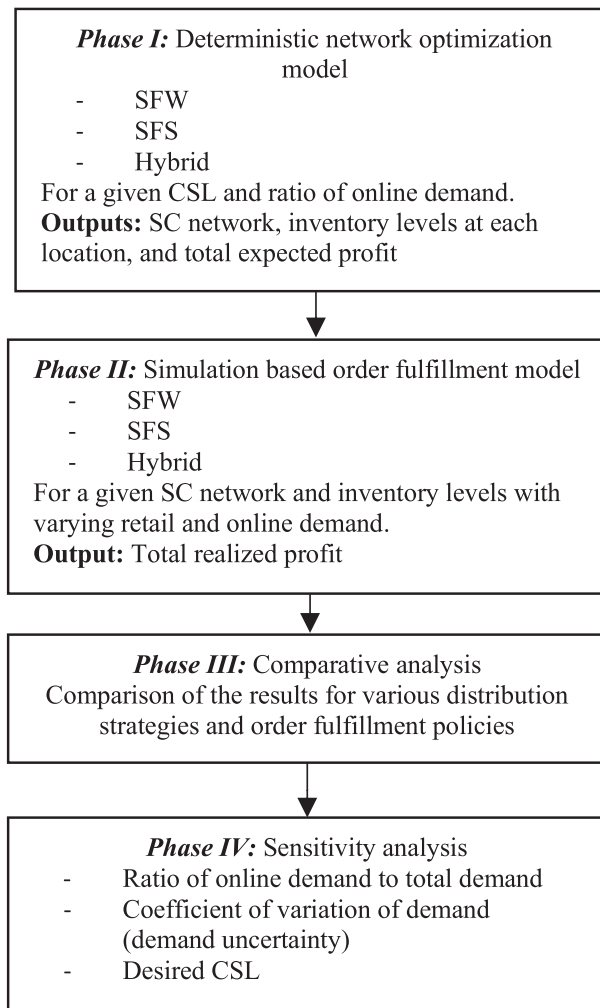


Fig. 2. The methodology followed in the study. CSL, customer service level; SC, supply chain; SFS, shipments from store; SFW, shipments from warehouse.

et al. (2021). In Phase I, a profit maximization model is defined to determine the optimal location of the facilities and level of inventories at each location to maximize expected profit. In this phase, facility locations are determined (Arslan et al., 2021), and inventory allocations are carried out (Lim et al., 2021). The developed mixed integer programming (MIP) model is run for three different channel design strategies for a given CSL, which is defined by the average demand and the desired number of the standard deviation of the demand distribution (Hovelaque et al., 2007). In this phase, total online and retail demand in the market is assumed to be equal to mean demand. However, the total shipments to the markets and the stores are forced to ensure a given CSL by adding the required level of safety inventory to the total number of products available at each store point as calculated in Equations (31) and (32). The required level of safety inventory depends on both

desired CSL and the demand variation. The details of the optimization model are provided in the mathematical formulation and notations section. The primary outputs of Phase I include the number, location, and capacity of the selected warehouses, total profit, and the inventory level at each location (warehouse or RS or both). These are used as inputs in the simulation model in Phase II.

In Phase II of the model, the fulfillment of the retail and online demand, both of which are stochastic in each market, is optimized to maximize the total profit (realized profit). To investigate the impacts of various order fulfillment policies, first, the shipments are forced to follow the selected distribution strategy (SFW, SFS, or hybrid strategy). Next, the market assignments for online orders are removed to see how the results are affected. Finally, the inventory levels provided by Phase I are used in a dynamic assignment as proposed by Mahar et al. (2009). The simulation model is run for 100 replications, and the demand data (following the normal distribution defined for each market for both customer segments) are randomly generated for each replication. All replications in Phase II are used at the same warehouse locations and inventory levels defined in Phase I.

In Phase III, several calculations and comparisons are conducted to draw more managerial insights and conclusions. In the last phase (Phase IV), the following sensitivity analysis is conducted to compare the results for various parameters: the ratio of online demand to total demand, the standard deviation of the demand to average demand (covariance), and the desired CSL. The objective of these analyses is to analyze how various distribution channel strategies and fulfillment policies react to changes in those parameters.

3.3. Model formulations and notations

This section provides the mathematical notations and formulations.

3.3.1. Strategic-level distribution network optimization model (deterministic)

The deterministic omnichannel facility location problem in the first phase is defined as a mixed integer programming problem adapted from Millstein et al. (2022). Let $i, j, s,$ and k be indices for the markets, potential warehouse locations, store locations, and warehouse capacities, respectively. Each market includes online demand and retail demand, and stores and warehouses are located at markets, so $J \subseteq I$ and $S \subseteq I$. Both online demand and retail demand are normally distributed with the means of μ_o and μ_r , respectively, and the standard deviations of σ_o and σ_r , respectively. The mean online demand μ_{oi} and the mean retail demand μ_{ri} are computed outside the optimization by using relevant demand data and an online demand ratio $OLpct$. Note that the model used in this study optimally determines which warehouse(s) (WH(s)) will serve the RS(s) in market i and which store or warehouse (depending on the strategy) will serve the online demand in market i (and thus the amount of demand served).

To model the warehouse options and costs, let γ_j^k be the capacity in units stored of a warehouse of size k located at j . The associated costs include $C_{h_wh_j^k}$, the inventory holding cost per unit for all units stored at the warehouse size k located at j ; C_{p_wh} , the fixed warehouse handling cost per unit for all units of online demand shipped from a warehouse; and $C_{f_wh_j^k}$, the fixed overhead cost of a warehouse size k located at j . $C_{f_wh_j^k}$ includes rent, utilities, and labor. There are also costs associated with order fulfillment from stores. These include C_{h_rs} , the inventory holding cost per

unit for RS for all units sold to either online customers or retail customers, and C_{p_rs} , the handling cost per unit of fulfilling online orders from RS in the SFS or hybrid channel design. Finally, there are the following three types of shipping costs modeled in the various shipment options: shipping cost to serve online demand from a warehouse, shipping cost to serve online demand from a store, and shipping cost to replenish the stores from a warehouse.

The complete set of parameters used in the strategic network optimization model is shown in Table 1.

Please note that the model for shipments from warehouse (SFW) is presented first, and then models for shipments from store (SFS) and the hybrid design are presented.

SFW formulation

In the SFW model, all online orders are shipped from a warehouse, and all RSs are replenished from a warehouse. The SFW model uses the following sets of decision variables:

$$Y_j^k, X_{ij}, Z_{ij}, V_{sj}, \text{ and } Q_{si}.$$

The objective of the model is to maximize total profit while fulfilling the mean demand at customer locations when it is profitable. Total profit is calculated as the difference between total revenue and total costs. Total costs include the costs for transporting products, holding them in inventory, handling online orders, and operating the warehouse. The handling costs only include those directly related to online order fulfillment (e.g., packing the box, creating the shipping manifest and labels, and labeling parcels for online orders).

The objective function for SFW is as follows:

Maximize Total Profit

$$\begin{aligned} &= p * \left(\sum_{s \in S} \sum_{i \in I} Q_{si} + \sum_{i \in I} \sum_{j \in J} Z_{ij} \right) - \left(\sum_{j \in J} \sum_{k \in K} C_{f_wh}^k * Y_j^k \right) - \left(\sum_{i \in I} \sum_{j \in J} C_{p_wh} * Z_{ij} \right) \\ &- \left(\sum_{j \in J} \left[\sum_{k \in K} C_{h_wh}^k * Y_j^k * \left[\sum_{i \in I} Z_{ij} \right] \right] \right) - \left(C_{h_rs} * \sum_{s \in S} \sum_{i \in I} Q_{si} \right) \\ &- \left(\sum_{s \in S} \sum_{i \in I} C_{t_sr} * Q_{si} \right) - \left(\sum_{i \in I} \sum_{j \in J} C_{t_sow} * Z_{ij} \right) - \left(\sum_{s \in S} \sum_{j \in J} C_{t_srw} * V_{sj} \right) \quad (1) \end{aligned}$$

The first term is the gross profit earned from retail and online sales. The second term is the fixed facility cost for opening facilities. The third term is the warehouse handling cost for online orders, which depends on the total number of units shipped from the warehouse to customers for all online demand (the Z_{ij} variables). The fourth term is the warehouse inventory holding cost, which depends on the number of units shipped to fulfill online orders (the Z_{ij} variables). The fifth term is the RS inventory holding cost. The last three terms in the objective function are the shipping costs for fulfilling retail demand, the shipping cost for fulfilling online demand, and resupplying RSs. The shipping cost for retail demand is used to make sure no retail demand is fulfilled by a retail store in

Table 1
Notation for the strategic-level distribution network optimization model.

Notation	Description
Sets:	
I	The set of nodes representing markets, each of which is also an online demand node
J	The set of candidate (potential) warehouse locations, which belong to the set of markets, $J \subseteq I$
S	The set of store locations, which belong to the set of markets, $S \subseteq I$
K	The set of warehouse sizes (capacities)
Parameters:	
P	Gross profit per unit (order)
λ_s	Capacity in number of units for a retail stores (RS) located at s
γ_j^k	Capacity in number of units for a warehouse of size k located at j
$C_{h_wh_j^k}$	Warehouse inventory holding cost per unit for warehouse of size k located at j
C_{p_wh}	Warehouse handling cost per unit of online demand served from a warehouse
$C_{f_wh_j^k}$	Warehouse fixed cost for warehouse of size k located at j
C_{h_rs}	Per unit inventory holding cost for RS located at s
C_{p_rs}	Fixed handling cost of online orders for RSs
μo_i	Mean online demand in market i
μr_i	Mean retail demand in market i
A_i	Requested response time in market i (miles in distance)
$covo_i$	Standard deviation of online demand to mean online demand (covariance) in market i
$covr_i$	Standard deviation of retail demand to mean retail demand (covariance) in market i
$z o_i$	z -value for online demand in market i (to ensure the desired CSL [customer service level])
$z r_i$	z -value for retail demand in market i (to ensure the desired CSL)
$OLpct_i$	Online demand as a percentage of total demand in market i
$C_{t_sow_{ij}}$	Shipping cost per unit for an online order to market i from a warehouse located at j
$C_{t_sos_{is}}$	Shipping cost per unit for an online order to market i from a store located at s
$C_{t_sr_{s_{is}}}$	Shipping cost per unit for a retail order to market i from a store located at s (set very high not to allow retail sales from stores in different geographical regions)
$C_{t_srw_{sj}}$	Shipping cost per unit to store s from a warehouse located at j
M	A very large number
Decision variables:	
Y_j^k	1 if a warehouse of size k is open at location j , 0 otherwise
X_{ij}	1 if online market i is served by warehouse j , 0 otherwise
L_{si}	1 if online market i is served by RS s , 0 otherwise
Z_{ij}	Total number of units shipped from a warehouse located at j to online market i to fulfill online demand
V_{sj}	Total number of units shipped to store at $s \in S$ from a warehouse located at $j \in J$
Q_{si}	Total number of units sold to RS customers from the store at s to market i (allowed only in the same geographical location by setting the relevant transportation cost very high)
U_{si}	Total number of units shipped to online customers in market i from a RS at s .

a different geographical region. The distance in the same geographical location (market) is also set to “0.” The constraints for the SFW model are as follows:

$$\sum_{j \in J} Z_{ij} \leq \mu o_i, \quad \text{for all } i \quad (2)$$

$$\sum_{j \in J} X_{ij} = 1, \quad \text{for all } i \quad (3)$$

$$X_{ij} \leq \sum_{k \in K} Y_j^k, \quad \text{for all } i \text{ and } j; \quad (4)$$

$$Z_{ij} \leq X_{ij} * \mu o_i, \quad \text{for all } j \text{ and } i \quad (5)$$

$$X_{ij} * dis_{ij} \leq A_i, \quad \text{for all } j \text{ and } i \quad (6)$$

$$\sum_{k \in K} Y_j^k \leq 1, \quad \text{for all } j; \quad (7)$$

$$\sum_{i \in I} (Z_{ij} * (1 + z * covo_i)) + \sum_{s \in S} (V_{sj} * (1 + z * covr_i)) \leq \sum_{k \in K} \gamma_j^k * Y_j^k, \quad \text{for all } j \quad (8)$$

$$\sum_{i \in I} Q_{si} \leq \lambda_s, \quad \text{for all } s \quad (9)$$

$$\sum_{s \in S} Q_{si} \leq \mu r_i, \quad \text{for all } i \quad (10)$$

$$\sum_{i \in I} Q_{si} \leq \sum_{j \in J} V_{sj}, \quad \text{for all } s \quad (11)$$

$$X_{ij}, Y_j^k \in \{0, 1\} \quad (12)$$

$$Z_{ij}, V_{sj}, Q_{si} \geq 0 \quad (13)$$

Equation (2) ensures that the units shipped from warehouses to online markets are less than or equal to the mean demand for each market. Altogether, Equations (3) to (6) ensure that only one open warehouse is assigned to each online market, online orders are fulfilled from that warehouse, and that warehouses can fulfill the orders within the requested response time. Equation (7) ensures that each warehouse is open at only one capacity (if open). Equation (8) ensures that the total shipment from warehouses is constrained by the selected capacities of the warehouses. Warehouse capacity should be sufficient for the desired CSL; z is the corresponding value from the z -distribution table satisfying the desired CSL. Equation (9) ensures that the total shipment from RS is constrained by the store capacity. Equation (10) ensures that the total number of products shipped to each market for retail orders is less than or equal to the mean retail demand. Equation (11) ensures that the number of units sold at the store does not exceed the number of units shipped from the warehouse to the store. Equation (12) is the binary domain constraint. Equation (13) is the non-negativity constraint.

Objective (1) has a nonlinear term for the warehouse holding cost as the binary warehouse size and location variable (Y_j^k) is multiplied. To linearize the objective, we define a positive variable

WHC_j^k as the warehouse holding cost for a warehouse of size k at location j , and then replace the nonlinear term in the objective with $\sum_{j \in J} \sum_{k \in K} WHC_j^k$ and add the following constraint:

$$C_{h_wh_j^k} * \left[\sum_{i \in I} Z_{ij} \right] - M (1 - Y_j^k) \leq WHC_j^k \quad \text{for all } j \text{ and } k. \quad (14)$$

The constant M in (14) is set to be higher than the sum of the maximum possible inventory holding cost of a warehouse. The full MIP formulation for the SFW model is then as follows:

Maximize Total Profit

$$\begin{aligned} &= p * \left(\sum_{s \in S} \sum_{i \in I} Q_{si} + \sum_{i \in I} \sum_{j \in J} Z_{ij} \right) - \left(\sum_{j \in J} \sum_{k \in K} C_{f_wh_j^k} * Y_j^k \right) - \left(\sum_{i \in I} \sum_{j \in J} C_{p_wh} * Z_{ij} \right) \\ &\quad - \left(\sum_{j \in J} \sum_{k \in K} WHC_j^k \right) - \left(C_{h_rs} * \sum_{s \in S} \sum_{i \in I} Q_{si} \right) - \left(\sum_{s \in S} \sum_{i \in I} C_{t_srSis} * Q_{si} \right) \\ &\quad - \left(\sum_{i \in I} \sum_{j \in J} C_{t_sowij} * Z_{ij} \right) - \left(\sum_{s \in S} \sum_{j \in J} C_{t_srw_{sj}} * V_{sj} \right) \end{aligned} \quad (15)$$

SFS formulation

In the SFS model, all online orders are shipped from a store, and all RSs are replenished from a warehouse. This requires the following new set of flow variables from stores to online markets: U_{si} and L_{si} .

The objective function for SFS is similar to that for SFW, but the warehouse handling cost is replaced with the RS handling cost, and online orders inventory holding costs in warehouses are replaced with online orders inventory holding costs in stores as follows:

Maximize Total Profit

$$\begin{aligned} &= p * \left(\sum_{s \in S} \sum_{i \in I} Q_{si} + \sum_{i \in I} \sum_{s \in S} U_{is} \right) - \left(\sum_{j \in J} \sum_{k \in K} C_{f_wh_j^k} Y_j^k \right) - \left(\sum_{i \in I} \sum_{s \in S} C_{p_rs} * U_{si} \right) \\ &\quad - \left(\sum_{s \in S} \sum_{i \in I} [Q_{si} + U_{si}] * C_{h_rs} \right) - \left(\sum_{s \in S} \sum_{i \in I} C_{t_srSis} Q_{si} \right) \\ &\quad - \left(\sum_{i \in I} \sum_{s \in S} C_{t_sos_{is}} * U_{si} \right) - \left(\sum_{s \in S} \sum_{j \in J} C_{t_srw_{sj}} * V_{sj} \right) \end{aligned} \quad (16)$$

subject to (7) and (10) with the following constraints:

$$\sum_{s \in S} U_{si} \leq \mu o_i, \quad \text{for all } i \tag{17}$$

$$\sum_{s \in S} L_{si} = 1, \quad \text{for all } i \tag{18}$$

$$U_{si} \leq L_{si} \times \mu o_i, \quad \text{for all } s \text{ and } i; \tag{19}$$

$$L_{si} * dis_{si} \leq A_i, \quad \text{for all } s \text{ and } i; \tag{20}$$

$$\sum_{s \in S} V_{sj} * (1 + (z * cov_{oi})) \leq \sum_{k \in K} \gamma_j^k Y_j^k, \quad \text{for all } j \tag{21}$$

$$\sum_{i \in I} (Q_{si} + U_{si}) \leq \lambda_s, \quad \text{for all } s \tag{22}$$

$$\sum_{i \in I} (Q_{si} + U_{si}) \leq \sum_{j \in J} V_{sj}, \quad \text{for all } s \tag{23}$$

$$L_{si}, Y_j^k \in \{0, 1\} \tag{24}$$

$$U_{si}, V_{sj}, Q_{si} \geq 0 \tag{25}$$

Constraints (17), (18), (19), (20), (21), (22), (23), (24), and (25) are analogous to constraints (2), (3), (5), (6), (8), (9), (11), (12), and (13) for SFW, respectively.

Hybrid model formulation

In the hybrid model, online orders are shipped from either a warehouse or an RS. RSs are replenished from a warehouse. The objective function for the hybrid model is as follows:

Maximize Total Profit

$$\begin{aligned} &= p * \left(\sum_{s \in S} \sum_{i \in I} Q_{si} + \sum_{i \in I} \sum_{j \in J} Z_{ij} + \sum_{i \in I} \sum_{s \in S} U_{is} \right) - \left(\sum_{j \in J} \sum_{k \in K} C_{-f_wh}^k Y_j^k \right) \\ &\quad - \left(\sum_{i \in I} \sum_{j \in J} C_{-p_wh} * Z_{ij} \right) - \left(\sum_{i \in I} \sum_{s \in S} C_{-p_rs} * U_{si} \right) - \left(\sum_{j \in J} \sum_{k \in K} WHC_j^k \right) \\ &\quad - \left(\sum_{s \in S} \sum_{i \in I} [Q_{si} + U_{si}] * C_{-h_rs} \right) - \left(\sum_{s \in S} \sum_{i \in I} C_{-t_sr} s_{is} * Q_{si} \right) - \left(\sum_{i \in I} \sum_{j \in J} C_{-t_sow} ij * Z_{ij} \right) \\ &\quad - \left(\sum_{s \in S} \sum_{j \in J} C_{-t_srw} sj * V_{sj} \right) - \left(\sum_{i \in I} \sum_{s \in S} C_{-t_sos} is U_{si} \right) \tag{26} \end{aligned}$$

Subject to (7), (8), (10), (14), (22), and (23) with the following constraints:

$$\sum_{s \in S} U_{si} + \sum_{j \in J} Z_{ij} \leq \mu o_i, \quad \text{for all } i \quad (27)$$

$$dis_{ij} - A_i \leq M * (1 - sf1_{ij}), \quad \text{for all } i \text{ and } j \quad (28.a)$$

$$Z_{ij} \leq M * sf1_{ij} \quad \text{for all } i \text{ and } j \quad (28.b)$$

$$dis_{si} - A_i \leq M * (1 - sf2_{si}), \quad \text{for all } i \text{ and } s \quad (28.c)$$

$$U_{si} \leq M * sf2_{si}, \quad \text{for all } i \text{ and } s \quad (28.d)$$

$$Z_{ij}, U_{si}, V_{sj}, Q_{si} \geq 0 \quad (29)$$

$$sf1_{ij}, sf2_{si}, Y_j^k \in \{0, 1\} \quad (30)$$

Constraint (27) is analogous to constraints (2) for SFW and (17) for SFS. Equations (28.a), (28.b), (28.c), and (28.d) altogether ensure that online orders can only be fulfilled from fulfillment centers (either WH or RS) within the requested response time. Constraint (29) is the non-negativity constraint. Equation (30) is the binary domain constraint.

For all three models, the following parameters are also calculated after the decision variables are defined by the relevant models:

RS inv_s = Total inventory kept available at RS *s*. It is the sum of the products shipped from a store to all markets for both online and retail demand and calculated by the following formula:

$$RS \text{ inv}_s = \sum_{i \in I} [Q_{si} * (1 + z * covr_i) + U_{si} * (1 + z * covo_i)], \quad \text{for all } s \quad (31)$$

WH inv_j = Total inventory kept available at warehouse *j*. It is the sum of the products shipped from warehouses to all markets for online demand and calculated by the following formula:

$$WH \text{ inv}_j = \sum_{i \in I} Z_{ij} * (1 + z * covo_i), \quad \text{for all } j \quad (32)$$

Please note that the safety stock is also included in the calculation of the available inventory at warehouses and stores depending on the desired CSL.

3.3.2. Simulation-based order fulfillment model

In the order fulfillment model, optimal values of decision variables ($\bar{Y}_j^k, \bar{V}_{sj}$) from the strategic network optimization model are used as input variables to fix the location and capacity of warehouses. Using the Monte Carlo simulation method following the normal distribution, the demand data are fixed for each replication of each distribution channel strategy. Then, various order fulfillment policies including the dynamic order assignment model are run to maximize the total profit. The

objective function and the constraints for the dynamic assignment model (allowing shipments from any warehouse or RS concurrently) are provided below.

The order fulfillment model uses the following sets of decision variables: Z_{ij} , Q_{si} , and U_{si} . The following parameters are also included in the model:

$RS\ inv_s$ = total inventory kept available at RS s .

$WH\ inv_j$ = total inventory kept available at warehouse j .

\overline{Y}_j^k = binary value of the variable provided by the network optimization model.

\overline{Do}_i = online demand calculated separately for each replication by the Monte Carlo simulation model following the normal distribution with a mean of μo_i and a std. dev. of σo_i .

\overline{Dr}_i = retail demand calculated separately for each replication by the Monte Carlo simulation model following the normal distribution with a mean of μr_i and a std. dev. of σr_i .

After the sampling is done for the online and the retail demand following the normal distribution, potential negative values are changed to the maximum value of “zero” and “sample value” to prevent any potential negative values for each replication.

The objective function for the simulation model is as follows:

Maximize Total Profit

$$\begin{aligned}
 &= p * \left(\sum_{s \in S} \sum_{i \in I} [Q_{si} + U_{si}] + \sum_{i \in I} \sum_{j \in J} Z_{ij} \right) - \left(\sum_{j \in J} \sum_{k \in K} C_{f_wh_j^k} * \overline{Y}_j^k \right) \\
 &+ \left(\sum_{i \in I} \sum_{s \in S} C_{p_rs} * U_{si} \right) - \left(\sum_{i \in I} \sum_{j \in J} C_{p_wh} * Z_{ij} \right) - \left(\sum_{s \in S} [Q_{si} + U_{si}] * C_{h_rs} \right) \\
 &- \left(\sum_{j \in J} \left[\sum_{k \in K} C_{h_wh_j^k} \overline{Y}_j^k * \left[\sum_{i \in I} Z_{ij} \right] \right] \right) - \left(\sum_{s \in S} \sum_{i \in I} C_{t_sr_sis} * Q_{si} \right) \\
 &- \left(\sum_{i \in I} \sum_{j \in J} C_{t_sow_{ij}} * Z_{ij} \right) - \left(\sum_{s \in S} \sum_{j \in J} C_{t_srw_{sj}} * \overline{V}_{sj} \right) - \left(\sum_{i \in I} \sum_{s \in S} C_{t_sos_{si}} U_{si} \right). \quad (33)
 \end{aligned}$$

The first term is the profit earned from retail and online sales. The second term is the facility cost for opening warehouses. The third term is the RS handling costs for online orders. The fourth term is the warehouse handling cost for online orders. The fifth term is the sum of RS inventory holding costs. The sixth term is the sum of the warehouse inventory holding costs. Because Y is fixed from Phase I, there is no nonlinear term in the inventory holding calculation. The last four terms in the objective function are the shipping costs for fulfilling retail demand, the shipping costs for fulfilling online demand from a warehouse, the shipping costs for resupplying RSs, and the shipping costs for fulfilling online demand from RSs.

The constraints for the order fulfillment model are as follows:

$$\sum_{j \in J} Z_{ij} + \sum_{s \in S} U_{si} \leq \overline{Do}_i, \quad \text{for all } i \quad (34)$$

$$\sum_{s \in S} Q_{si} \leq \overline{Dr}_i, \quad \text{for all } i \quad (35)$$

$$\sum_{i \in I} [Q_{si} + U_{si}] \leq R\text{Sinv}_s, \quad \text{for all } s \quad (36)$$

$$\sum_{i \in I} Z_{ij} \leq W\text{Hinv}_j, \quad \text{for all } j \quad (37)$$

$$dis_{ij} - A_i \leq M * (1 - sf1_{ij}), \quad \text{for all } i \text{ and } j \quad (38.a)$$

$$Z_{ij} \leq M * sf1_{ij}, \quad \text{for all } i \text{ and } j \quad (38.b)$$

$$dis_{si} - A_i \leq M * (1 - sf2_{si}), \quad \text{for all } i \text{ and } s \quad (38.c)$$

$$U_{si} \leq M * sf2_{si}, \quad \text{for all } i \text{ and } s \quad (38.d)$$

$$Z_{ij}, U_{si}, Q_{si} \geq 0 \quad (39)$$

$$sf1_{ij}, sf2_{si} \in \{0, 1\} \quad (40)$$

Equation (34) ensures that the units shipped from warehouses or RSs to online markets are less than or equal to online demand for each market. Equation (35) ensures that the total number of products shipped to each market to fulfill retail orders is less than or equal to retail demand. Equation (36) limits the number of units sold at the store or shipped to fulfill online orders to the number of units available at the store. Similarly, Equation (37) limits the total number of units shipped from each warehouse to fulfill online orders to the available inventory at each warehouse. Altogether, Equations (38.a), (38.b), (38.c), and (38.d) ensure that online orders can only be fulfilled from fulfillment centers that are able to fulfill the requested response time. Equation (39) is the non-negativity constraint. Equation (40) is the binary domain constraint.

The simulation model presented above assumes that the firm is already applying the hybrid distribution network strategy presented above. Depending on the order fulfillment policy and applied network optimization strategy, the model might require some adjustments. In other words, if the firm is operating under the SFW strategy, then the model needs to be revised so that the inventory available at warehouses is used to fulfill online orders and the inventory available at RSs is used to fulfill retail orders. Also, depending on the applied fulfillment policy, warehouse and customer assignments or RS and customer assignments may be forced. The details of the fulfillment policies for each experiment are specified in the numerical analysis section.

4. Numerical examples and results

To analyze and illustrate the results, the Daskin dataset based on real geographic locations and population distributions is used. The dataset is based on 49 nodes representing the 48 state capitals of the contiguous United States, plus Washington, DC (from <https://daskin.engin.umich.edu/software/>). The Daskin dataset defines the demand for each node as proportional to the corresponding state's population. Shipping distances are included in the dataset (Daskin, 1995). For this study, a store is located at each market (specifically, in each state). Store capacity is defined as 20 million units, which is equal to the lowest capacity level for the warehouse choices. Eight locations are selected as candidate warehouse locations. These are chosen to avoid candidate locations nearby.

To set the relevant cost parameters, a prototypical product is considered. Table 2 provides the values of several parameters. Five different warehouse capacity choices are modeled, and the parameters are differentiated for each capacity choice. Inventory holding and warehouse overhead costs demonstrate economies of scale as there are different warehouse and store order handling costs defined. Note that the value of the retail inventory holding cost is set higher than the warehouse holding cost because, typically, retail space is more expensive to build, maintain, and rent. Store handling cost is defined as higher than warehouse handling cost (Lang and Bresoles, 2013; Hubner et al., 2016; Hubner et al., 2022; Millstein et al., 2022). That is also supported by many practical examples (e.g., *Total Hockey*, *Weekends Only*, etc.). Profit and cost parameters are adopted from Millstein et al. (2022). In their study, the authors analyzed five model products to see how the results change through different product types. For the base scenario, the requested response time is set as “one day.”

Parcel shipping costs for online orders are calculated using a cost equation analogous to the cost equation estimated by Jasin and Sinha (2015) and utilized by Govindarajan et al. (2021). The parameters in the equation are updated by 2022 United Parcel Service (UPS) ground shipping rates for an item weighing 1 pound: the online shipping cost from warehouse to online market (c_{t_sowij}) or from RS to online market ($c_{t_sos_{is}}$) = $10.08 + 0.00092dis_{ij}$ (or dis_{is}), where dis_{ij} (or dis_{is}) is the distance in miles from fulfillment point to online market i . Inbound transportation costs ($c_{t_srw_{sj}}$) are adjusted assuming 10 products are bundled in one shipment to exploit the economies of scale.

Table 2
Profit and cost parameters for prototypical product.

Profit/cost parameter	Value of the parameter
Profit per product— p (\$)	10
RS handling costs per product— C_{p_rs} (\$)	1.10
RS inv. holding costs per product— C_{h_rs} (\$)	1.65
Warehouse (WH) handling cost per product— C_{p_wh} (\$)	1.0
WH inv. holding costs per product— $C_{h_wh_j^k}$ (\$) capacity level bw (20–240 M):	(1.65–0.65)
Annual WH cost— $C_{f_wh_j^k}$ (\$) capacity level bw (20–240 M):	77,500,000–260,000,000

Table 3
Deterministic model results for various ratios of online demand under different distribution network strategies.

		Online demand ratio				
		20%	40%	50%	60%	80%
Total Profit (million \$)	SFW	805.2	798.2	801.0	808.8	839.1
	SFS	809.1	817.2	812.1	807.0	778.2
	Hybrid	859.8	919.6	940.4	961.0	998.3
Total Online (OL) Sales (million units)	SFW	20.9	79.2	99.0	130.9	174.6
	SFS	49.4	98.8	123.6	148.2	193.6
	Hybrid	49.4	98.8	123.6	148.2	197.6
OL Dem. Fill Rate (ODFR)	SFW	42.2%	80.2%	80.2%	88.3%	88.3%
	SFS	100.0%	100.0%	100.0%	100.0%	98.0%
	Hybrid	100.0%	100.0%	100.0%	100.0%	100.0%
Total Retail Sales (million units)	SFW	191.9	148.2	123.6	98.8	49.4
	SFS	193.8	148.2	123.6	98.8	49.4
	Hybrid	193.8	148.2	123.6	98.8	49.4
Retail Demand Fill Rate (RDFR)	SFW	97.1%	100.0%	100.0%	100.0%	100.0%
	SFS	98.1%	100.0%	100.0%	100.0%	100.0%
	Hybrid	98.1%	100.0%	100.0%	100.0%	100.0%

4.1. Results for the deterministic strategic level network optimization model (Phase I)

First, the strategic network optimization model is solved to determine the optimal location of the facilities and the traffic among the nodes of the SC that maximize the expected profit under various distribution channel design strategies (SFW, SFS, and hybrid) assuming that mean demand is the actual demand on the market.

Table 3 depicts the results for various online demand ratios for different distribution network strategies when the coefficient of variation (standard deviation of the demand/average demand) equals 0.10, the desired CSL is 90% (z -value = 1.28), and the requested response time is one day (500 miles in distance).

The results show that the hybrid strategy outperforms the other two strategies as expected. This is in line with the study by Millstein et al. (2022). Their analysis shows that the hybrid strategy is always at the optimal choice. According to Table 3, the profit difference between the hybrid strategy and the SFS and SFW strategies may be explained by the flexibility of fulfilling the online demand from either a warehouse or RS. In the hybrid strategy, the online demand within a radius of 500 miles from warehouses may be fulfilled from warehouses, and online demand from other customer locations that do not have any warehouse within a radius of 500 miles may be fulfilled from RSs. As the online demand ratio increases, that flexibility becomes more important, and the profit difference increases.

Demand fill rate shows the fulfilled demand as a ratio of actual demand in the market. The online demand fill rate difference between the SFS and SFW strategies decreases as the online demand ratio increases. In SFW strategies, the system decides to open more warehouses to get closer to the markets. The results also show that the SFS strategy outperforms the SFW strategy in terms of total profit and demand fill rate when the online demand ratio is below 60%. That is mainly

due to being geographically closer to the markets in SFS strategies. In SFW strategies, the online demand fill rate is always lower because of not having a warehouse within a radius of 500 miles from some markets. As the online demand ratio increases, it becomes more profitable to open new warehouses. SFW (with four warehouses) outperforms SFS when the online demand ratio equals 80%. The SFW model decides to open only one warehouse when the online demand ratio equals 20%.

4.2. Results for the simulation-based order fulfillment models (Phase II)

In Phase II, the results of the deterministic models are used as inputs, and the order fulfillment models are run to investigate how the results change under different fulfillment strategies. The process is repeated for each channel design strategy, and the online demand ratio is given as 50% in all runs. First, the shipments are forced to follow the predetermined distribution strategy in the first phase. Then, warehouse and market assignments (for SFW) and RS and market assignments (for SFS) are removed to see how the total profit of the SC changes. Finally, predetermined network nodes and available inventories are used in a dynamic assignment process in which an e-fulfillment location with excess inventory may be able to fulfill the online sales. The models are run for multiple replications for scenarios generated by a Monte Carlo simulation model to generate statistics for each combination.

4.2.1. Simulated results for SFW

Here, the defined distribution network and inventories available at warehouses (for online demand) and RSs (for retail demand) are run through the simulation model to understand how the actual profits and sales change under various order fulfillment policies with various values of CSL (50%, 75%, 90%, and 95%). Note that the available inventories were defined in the deterministic phase to ensure the desired CSL. This is also reflected in the shipping and inventory holding costs. Table 4 depicts the results for three different fulfillments strategies: SFW respecting the warehouse and market assignments, SFW ignoring the warehouse and market assignments (i.e., allowing online shipments from any warehouse to any market whenever profitable and providing the requested response time), and a dynamic assignment policy that allows shipments from any warehouse or store (providing the requested response time) that has the available product. To give a notion of the variance and the worst and best-case performance, the values in the box-and-whisker plot representation of replication results and average (arithmetic mean) values are individually presented in the table.

Some findings can be drawn from the results. First, there is a significant difference between the expected profit (\$801 million) for the mean demand in the first phase and the average realized profit (around \$710 million) when the SFW strategy is applied even when the standard deviation of the demand is only 10% of the mean demand. Online sales do not decrease much because more than the average demand (depending on the CSL) is stored at warehouses, and the system tries to fulfill the demand as much as possible. Ignoring the warehouse and market assignments may help the firm increase its profits; however, its impact on profit is minimal (less than 0.1%). The second finding regards the dynamic assignment policy. This policy allows the model firm to increase

Table 4
Simulation model results for SFW strategy (CSL = 90%).

	Average values											
	Total profit (million \$)					Transport. costs (million \$)					Total retail sales (million units)	
	Box-and-whiskey values					Total Profit (million \$)					Total OL sales (million units)	
	Min	Q1	Median	Q3	Max	Total Profit (million \$)	Transport. costs (million \$)	Total OL sales (million units)	ODFR	Total retail sales (million units)	RDFR	
10 Reprs.												
Deterministic	801.0					1143.6	99.0	123.5	80.2%	100.0%		
SFW	631.1	694.8	720.1	731.6	754.7	710.8	98.9	117.2	80.4%	94.3%		
SFW-Without (WO) Assignment Const.	631.3	695.1	720.3	731.8	754.9	711.0	98.9	117.2	80.4%	94.3%		
100 Reprs.												
Dynamic Assignment	681.8	731.0	758.0	772.0	805.3	750.9	104.4	117.2	84.9%	94.3%		
SFW	631.1	691.6	709.3	724.0	764.8	707.5	99.1	116.6	80.1%	94.4%		
SFW-WO Assignment Const.	631.3	691.8	709.5	724.3	765.0	707.7	99.1	116.6	80.1%	94.4%		
1000 Reprs.												
Dynamic Assignment	681.8	735.9	751.8	765.3	805.3	750.4	104.9	116.6	84.9%	94.4%		
SFW	631.1	691.4	707.9	723.9	771.5	707.3	98.9	116.7	80.1%	94.5%		
SFW-WO Assignment Const.	631.3	691.6	708.2	724.2	771.6	707.6	98.9	116.7	80.1%	94.5%		
Dynamic Assignment	678.5	734.0	750.9	766.2	825.7	749.6	104.7	116.7	84.8%	94.5%		

Table 5
Simulation model results for the SFW strategy (CSL = 50%, 75%, 90%, and 95%).

	Total profit (average–million \$)				Total online sales (average–million units)			
	CSL = 50%	CSL = 75%	CSL = 90%	CSL = 95%	CSL = 50%	CSL = 75%	CSL = 90%	CSL = 95%
Deterministic	819.6	813.7	801.0	799.8	99.0	99.0	99.0	98.8
SFW	725.1	729.9	707.5	698.4	97.4	98.9	99.1	99.1
SFW–WO Assignment Const.	726.5	730.1	707.7	698.6	97.6	99.0	99.1	99.1
Dynamic Assignment	746.3	757.4	750.4	750.2	100.3	102.7	104.9	106.4

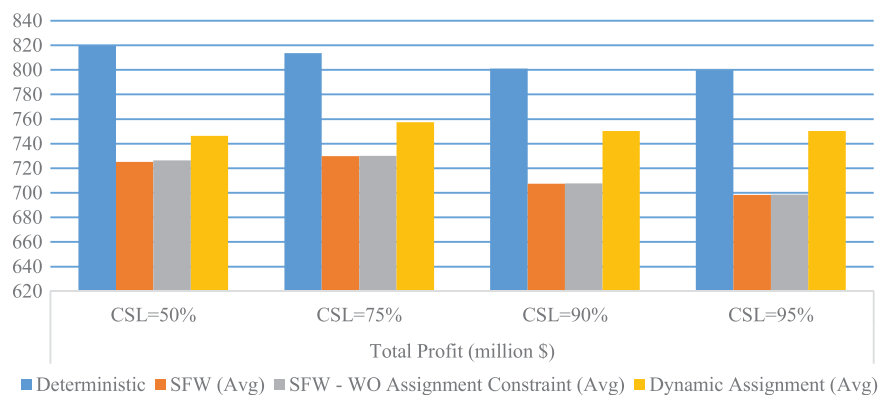


Fig. 3. Total profit for shipments from store (SFW) strategy under various order fulfillment policies. CSL, customer service level.

its profits substantially (approximately 6%), mainly due to increased online sales. Moreover, the total transportation costs increase because of the increased distance between the markets and the warehouses in which the product is available. The results also show that convergence with 100 replications is satisfactory. Therefore, simulation models are run for 100 replications from now on.

The same analysis is repeated for the desired CSL value of 50% (z -value: 0.0), 75% (z -value: 0.675), and 95% (z -value: 1.645; Table 5). In the table, the total profit of the deterministic model decreases as the service level increases. The profit decreases mainly because of the increased capacity requirements of the warehouses in the deterministic model. On the other hand, average profit in the simulation models increases from a CSL value of 50% to 75% due to getting closer to the optimal CSL. Beyond the CSL value of 75%, the average profit starts decreasing. A comparison of the performance of various strategies in the simulation model, however, indicates that the total profit reduction is even less for a dynamic assignment (Fig. 3) with a higher level of desired CSL. That is, the profit difference between the deterministic model (expected profit for the mean demand) and the dynamic assignment model decreases as the desired CSL increases.

In the base scenario, the requested response time is one day. That is, online demand must be fulfilled from any fulfillment center that is not farther than 500 miles. The same analysis is repeated

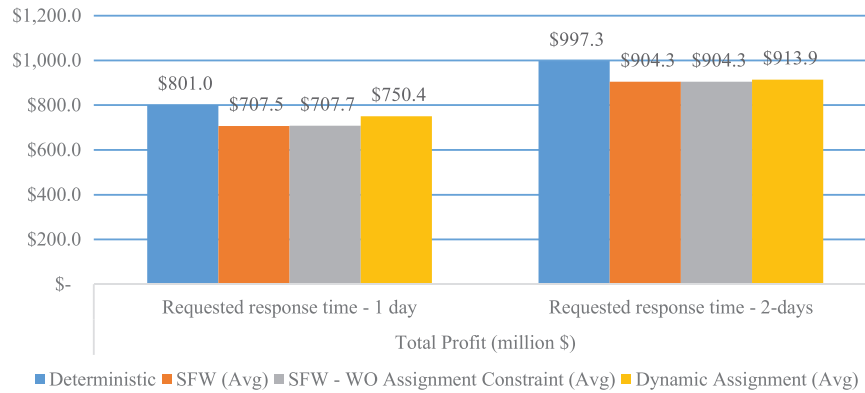


Fig. 4. Total profit for SFW strategy under different requested response time constraints.

to investigate how the results change when the requested response time is increased to two days (Fig. 4).

As expected, the relaxed constraint of fulfilling the demand from any warehouse within a radius of 1000 miles helped to increase profit. The expected profit increase is about 25% in the deterministic setting. However, additional profit generated by applying the dynamic assignment policy is only around 1% in the simulated model when the requested response time is two days. That ratio was about 6% when the requested response time was one day.

4.2.2. Simulated results for SFS

The same procedure is followed for the SFS distribution strategy. The distribution network decisions (selected warehouse locations and capacities) and inventory levels at RSs (no inventory is allowed at warehouses) defined in the deterministic model for the SFS are used in the simulation model for a given CSL (50%, 75%, 90%, and 95%).

Table 6 depicts the results of the following two different fulfillment strategies: SFS respecting the RS and market assignments from the deterministic model, and SFS ignoring the RS and market assignments (i.e., allowing online shipments from any RS to any customer within the allowed

Table 6
Simulation model results for the SFS strategy (CSL = 90%).

	Total profit (million \$)					Average values					
	Box-and-whiskey values					Total profit (million \$)	Transp. costs (million \$)	OL Sales (million units)	Ret. sales (million units)	ODFR	RDFR
	Min	Q1	Median	Q3	Max						
Deterministic	812.1						1496.0	123.5	100.0%	123.5	100.0%
SFS	489.4	558.1	577.3	588.7	633.3	574.4	1423.0	113.2	91.5%	113.1	91.6%
SFS–WO Assignment Constraint	567.2	630.2	651.4	665.7	706.5	647.0	1528.9	123.4	99.8%	113.1	91.6%

Table 7

Simulation model results for the SFS strategy (CSL = 50%, 75%, 90%, and 95%).

	Total profit (average–million \$)				Total online sales (average–million units)			
	CSL = 50%	CSL = 75%	CSL = 90%	CSL = 95%	CSL = 50%	CSL = 75%	CSL = 90%	CSL = 95%
Deterministic	878.7	822.5	812.1	794.9	116.5	123.5	123.5	123.5
SFS	683.0	614.3	574.4	534.9	101.0	112.0	113.2	113.3
SFS–WO Assignment Const.	725.8	690.7	647.0	607.5	107.0	122.8	123.4	123.6

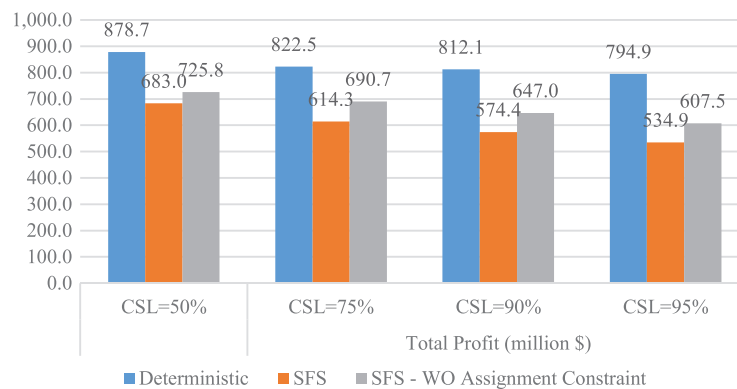


Fig. 5. Total profit changes for the shipments from store (SFS) strategy. CSL, customer service level.

distance). Because no inventory is held at warehouses, the dynamic fulfillment policy is not applied in this particular analysis.

The results show that the total profit difference between the deterministic model (expected profit for the mean demand) and the simulation model (average profit) is even higher under the SFS strategy. This is mainly due to higher inventory holding costs at RSs and the inability to fulfill the demand from warehouses. The same analysis is repeated for the service levels of 50% (z -value: 0.000), 75% (z -value: 0.675), and 95% (z -value: 1.645; Table 7 and Fig. 5).

Comparing the performance of various shipment policies in SFS strategies to the policies in SFW strategies indicates that, at a higher level of CSL, the total profit reduction is higher in SFS strategies than in SFW strategies. That is, the SFS strategy seems to perform better for certain ratios of online demand when the model is run for the mean demand. However, when the demand variation is taken into account, SFW-based strategies perform better, especially when the desired CSL is higher. Furthermore, dynamic assignment strategies may improve performance in SFW strategies.

In the base scenario, the requested response time is one day (500 miles). The same analysis is repeated to investigate how the results change when the requested response time is changed to two days (Fig. 6).

In the SFS strategy, online demand is mostly fulfilled from the store in the same city, as there is one RS in each city. Therefore, the change in the requested response time did not affect the total

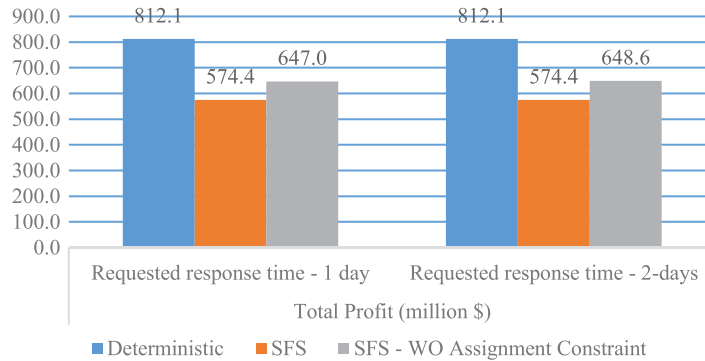


Fig. 6. Total Profit for SFS strategy under different requested response time constraints.

Table 8
Simulation model results for the hybrid strategy (CSL = 90%).

	Total profit (million \$)					Average values					
	Box-and-whisker values					Total profit (million \$)	Transp. costs (million \$)	OL sales (million units)	ODFR	Ret. sales (million units)	RDFR
	Min	Q1	Median	Q3	Max						
Deterministic	940.4						1433.9	123.5	100.0%	123.5	100.0%
Dynamic Assignment	715.9	777.9	796.1	808.2	854.5	794.0	1452.5	123.4	99.8%	112.9	91.4%

profit or the fulfilled online demand in deterministic and simulation-based settings. However, the total profit is increased when store-market assignments are removed, so the demand may be fulfilled from any store with the available product. The profit increase ratio of the removal of the market assignment constraint is 12.6% when the required response time is one day. That ratio slightly increases to 12.9% when the requested response time is relaxed to two days.

4.2.3. Simulated results for the hybrid strategy

The same procedure is applied to the hybrid distribution strategy. Only the results for one type of dynamic order fulfillment policy are included because the hybrid strategy is similar to the dynamic order assignment strategy. Table 8 depicts the detailed results of the model when CSL equals 90% (base scenario).

Then, the model is run for various values of CSL. Table 9 and Fig. 7 present the results. The results show that the changing pattern of total profit difference between the deterministic model and the simulation model is similar to that of the SFW model. This is because of the availability of the inventories at both warehouses and RSs in both strategies, which provides the flexibility of fulfilling the demand from RSs and warehouses in dynamic assignment policies.

Required response time analysis is also conducted for the hybrid strategy to explore how the results respond to the change in the required response time (Fig. 8). As expected, the relaxed constraint of fulfilling demand from any warehouse within a radius of 1000 miles helped increase profit

Table 9
Simulation model results for the hybrid strategy (CSL = 50%, 75%, 90%, and 95%).

	Total profit (million \$)				Total online sales (million units)			
	CSL = 50%	CSL = 75%	CSL = 90%	CSL = 95%	CSL = 50%	CSL = 75%	CSL = 90%	CSL = 95%
Deterministic	976.2	945.8	940.4	931.0	116.5	123.5	123.5	123.5
Dynamic Assignment (Avg.)	844.3	818.9	794.0	770.3	111.0	122.6	123.4	123.6

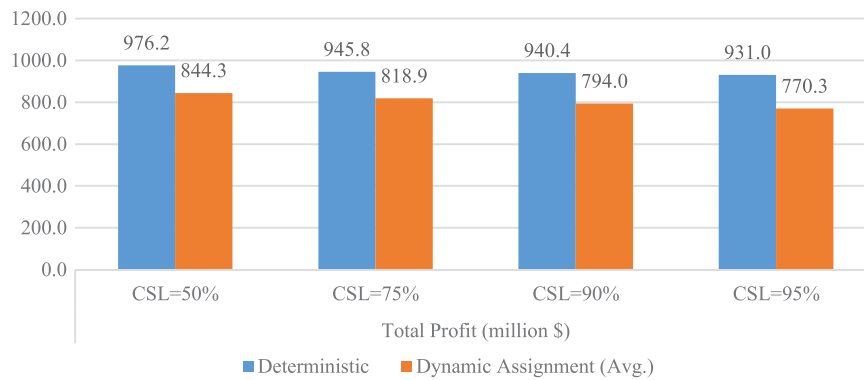


Fig. 7. Total profit changes for the hybrid strategy (deterministic vs. dynamic assignment model). CSL, customer service level.

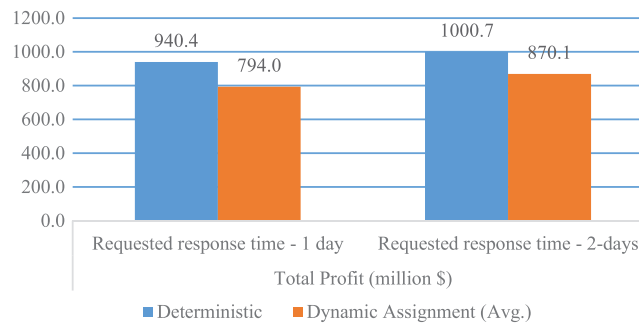


Fig. 8. Total profit for the hybrid strategy under different requested response time constraints.

in the hybrid strategy. The expected profit increase is 6.4% in the deterministic setting. However, the ratio for the simulation model setting is 9.6%. That means flexibility in fulfilling the demand from farther locations becomes more critical when demand uncertainty is considered.

4.2.4. Comparison of SFW, SFS, and hybrid strategies

In previous subsections, various distribution network strategies are individually investigated. Here, the results for those distribution network strategies are compared to draw some conclusions regarding, under what conditions, which distribution strategy should be adopted, and how demand

Table 10
Results for various network strategies (CSL = 50%, 75%, 90%, and 95%).

	Total profit (million \$)			
	CSL = 50%	CSL = 75%	CSL = 90%	CSL = 95%
SFW Deterministic	819.6	813.7	801.0	799.8
SFW Simulated (Avg)	725.1	729.9	707.5	698.4
SFW Dynamic Assignment (Avg)	746.3	757.4	750.4	750.2
SFS Deterministic	878.7	822.5	812.1	794.9
SFS Simulated (Avg)	683.0	614.3	574.4	534.9
SFS–WO Assignment Const. (Avg)	725.8	690.7	647.0	607.5
Hybrid Deterministic (Avg)	976.2	945.8	940.4	931.0
Hybrid Simulated (Avg)	844.3	818.9	794.0	770.3

Table 11
Results for various network strategies (requested response time = one and two days).

	Total profit (million \$)	
	Requested response time one day	Requested response time two days
SFW Deterministic	801.0	997.3
SFW Simulated (Avg)	707.5	904.3
SFW Dynamic Assignment (Avg)	750.4	913.9
SFS Deterministic	812.1	812.1
SFS Simulated (Avg)	574.4	574.4
SFS–WO Assignment Const. (Avg)	647.0	648.6
Hybrid Deterministic	940.4	1000.7
Hybrid Simulated (Avg)	794.0	870.1

uncertainty and requested response time affects the decision. The results for various distribution network strategies are summarized in Table 10, where the online demand ratio is equal to 50% and the requested response time is one day. One conclusion is that SFS seems to outperform SFW strategies in deterministic settings (except when $CSL = 95\%$). However, when demand uncertainty is considered, SFW outperforms SFS. The performance of the SFW strategy gets even better when the dynamic assignment policies are applied. The performance difference between SFW and SFS, in simulation models, gets even higher for higher values of CSL. The hybrid strategy always outperforms the SFS and SFW strategies. However, the profit difference between the hybrid strategy and other strategies gets decreased in simulation models. That profit difference gets even smaller for the higher values of the desired CSL. A detailed analysis based on the higher values of the coefficient of demand variance is also presented in the sensitivity analysis section.

Table 11 summarizes the total profit values of various distribution network strategies for the requested response times of one and two days when the online demand ratio is equal to 50% and CSL is 90%. The results show that total profit decreases in all distribution network strategy alternatives with a stricter requested response time as expected. However, profit reduction is highest for SFW and lowest for SFS. This is because online orders are mostly fulfilled by local stores in SFS. This

result indicates that having local stores fulfill online demand becomes more desirable for more strictly requested response times.

4.3. Sensitivity to the model parameters

The analysis of various distribution network strategies shows how expected and realized profit change for different values of CSL in a given demand variation and online demand ratio. In this section, how the results change with different online demand ratios and coefficients of variation is investigated.

4.3.1. Online demand ratio

First, developed models are used to investigate how the expected and realized profits change in response to the changes in the online demand ratio (20%, 40%, 60%, and 80%; Appendix 1). In this particular analysis, the desired CSL is given as 90%, the coefficient of variation of the demand (both for online and retail demand) is assumed to be 10%, and the required response time is one day.

The results for the SFW channel design strategy show that the total profit difference (between the expected total profit for the mean demand and average total profit for the simulation model) decreases as the online demand ratio to the total demand for the firm increases. Another finding is that the dynamic assignment policy is more beneficial to the firm as the online demand ratio increases.

The results for the SFS channel design strategy show that, unlike the results for the SFW strategy, the total profit difference (between expected total profit and realized total profit) does not decrease as the online demand ratio increases. However, the additional profit from the removal of the store-market assignment increases as the online demand ratio increases.

The results for the hybrid channel design strategy are somewhat similar to the results for the SFW strategy. The expected profit in the first phase seems to increase slightly as the online demand ratio increases. Meanwhile, the total profit of the simulation model increases more as the online demand ratio increases.

When the results for the SFW and hybrid strategies are compared, it can be concluded that the total profits in the simulation models are not very different, even though there seems to be a huge expected profit difference (from \$50 to \$160 million as the online demand ratio increases) for the deterministic model. This means that the hybrid model in the deterministic model seems to misleadingly generate higher results than firms might generate by fulfilling the online demand both from warehouses and RSs when the demand uncertainty is revealed.

4.3.2. The coefficient of variation

Developed models are also used to investigate how the total profit in the first and second phases (the expected and realized profits) change in response to the changes in the coefficient of variation (0.10, 0.20, 0.30, 0.40, and 0.50; Appendix 2) in both online demand and retail demand. In this analysis, the desired CSL, the online demand ratio, and the requested response times are given as 90%, 50%, and one day, respectively.

The results for the SFW channel design strategy show that the total expected profit in the deterministic setting decreases slightly as the variation of the demand increases due to increased capacity requirements at warehouses. Furthermore, the realized profit in the simulation model declines rapidly. At the high level of variation, the profit increase realized by the application of the dynamic assignment policy diminishes.

The results for the SFS strategy follow a similar pattern. However, the profit difference is higher in SFS, compared to SFW strategies. In SFS model, with a coefficient of variation of 0.5, the average profit in the simulation model (realized profit) may approach zero. However, the expected profit is calculated as approximately \$680 million at that level of variation when the variation in demand is ignored. SFS being profitable when demand uncertainty is ignored is misleading because if the firm understands the impact of the demand variation, it might decide not to be operative.

The results for the hybrid design strategy are somewhat similar to those for the SFW strategy. The expected profit in the deterministic setting slightly decreases as the coefficient of variation of demand increases. Meanwhile, the realized profit in the simulation model rapidly declines.

When the results for the SFW and hybrid design strategies are compared using a coefficient of demand variation value of 0.3 or lower, it can be concluded that the total realized profits in the simulation models are not very different, even though there is a substantial profit difference in the deterministic models. These findings reveal that when demand uncertainty increases on the market, having inventory available at both warehouses and RS becomes more important than the channel design strategy (either hybrid or SFW).

4.3.3. *The coefficient of variation of demand and the desired CSL*

How the total profit of the model firm (the expected and realized profits) changes in response to simultaneous changes to the coefficient of variation of demand and the desired CSL is also investigated. In this analysis, the online demand ratio and the requested response time are assumed to be 50% and one day, respectively.

Previous analyses already revealed that the realized profit decreases substantially as the coefficient of variation or the desired CSL is increased. The SF model results presented in Appendix 3 also show that the higher the CSL, the more critical role dynamic assignment policy plays as demand variation increases. From this figure, it is noticeable that the profit difference between the deterministic model and the dynamic assignment policy in the simulated model decreases at the desired CSL of 95%. That is, the slope of the total profit for the SFW model is higher than the slope of the dynamic assignment policy in the simulated model. The results for SFS channel design strategy in Appendix 3 show that the extent to which profit declines as variation increases is greater at a higher value of CSL. The profit increase caused by the removal of the RS and market assignment is still limited even at a higher level of variation and desired CSL. The results for the hybrid channel design strategy show that the total profit also rapidly declines as demand variation increases even though it has the full flexibility to fulfill the online demand. The profit decline is even greater when $CSL = 95\%$.

When the results of different distribution channel strategies in that particular experiment are compared, several conclusions can be drawn. First, the total profit in the simulation model at a higher level of variation is higher for the SFW than the hybrid strategy when the dynamic assignment policy is applied. This is due to the availability of more inventory at the warehouses in the

SFW model. When both the variation and the desired CSL are high, the likelihood of the product not being available at stores in proximity increases. That is, the average distance between the available store and the market increases. Therefore, shipping products from RS to markets become more expensive. In addition, the distance between the available store and the customer point may be further than 500 miles, which is the limitation to delivering the product in the requested response time. When the warehouses have more inventory available, they are better able to handle the increased demand variation, even with a high value of CSL. Also, the SFS model copes the least successfully with variation even though the results of the deterministic models show that the SFS model is more profitable than the SFW model in most cases. This is especially true when the desired CSL is high.

5. Conclusions and further research suggestions

This study aimed to investigate the effect of uncertain demand on various distribution channel design strategies and analyze the impacts of various online order fulfillment policies (more specifically, the dynamic assignment policy). To conduct the analysis, a two-staged network optimization model was built that accomplished the following: (a) a deterministic network optimization model determined the optimal distribution strategy for the mean demand, and (b) a simulation model determined the optimal network flow to maximize the total profit for a given scenario for uncertain demand.

Many conclusions and managerial insights can be drawn from the results of the analyses. First, the firms should take into account the demand variation, requested delivery speed, and desired product availability while designing their omnichannel distribution network. As illustrated in the study, there are changing results for the different values of coefficient of demand variation, requested delivery speed, and desired CSL. Second, the results show that the hybrid strategy in which the online shipments are fulfilled from either store or warehouse is always at the optimal choice. However, the profit difference between the hybrid strategy and SFW and SFS strategies may substantially change depending on changes in demand uncertainty, requested service speed, and desired CSL. Moreover, the hybrid strategy increases the complexity of distribution operations. Therefore, firms should analyze their omnichannel distribution network regarding demand uncertainty, requested service speed, and desired CSL to figure out when it is more critical to offer shipments both from warehouses and RSs for online demand. Another managerial insight is regarding the requested delivery time. Results show that total profit decreases in all distribution network strategies with a shorter requested delivery time. However, the profit reduction is highest for SFW strategies and lowest for SFS strategies. That means having local stores fulfill online demand becomes more desirable for stricter requested delivery times.

In a deterministic setting, the hybrid strategy is always at the optimal choice, and the SFS strategy seems to outperform the SFW strategy in most scenarios. Moreover, the simulated results (which consider demand uncertainty) indicate that the actual total profit might be quite lower than what is expected in the deterministic settings, even when demand variation and desired CSL are quite low. As variation and desired CSL increase, the realized profit declines sharply. SFW and hybrid network design strategies better cope with demand variation because the profit decline in the simulation model is more in SFS strategies. The simulated results in Phase II also show that not respecting

the warehouse and market assignments (in the SFW strategy) and RS to market assignments (in the SFS strategy) might help firms handle variation; however, the impact on the total profit is minimal. Furthermore, a dynamic assignment successfully copes with demand variation. The results of the dynamic assignment policy indicate the importance of having the products available in central repositories to cope with demand variation. The results also show that SFS being more profitable in the deterministic setting is misleading because it becomes less profitable (compared to the SFW strategy) when demand variation is taken into consideration. That is true even when there is a low level of demand variation and the desired CSL is low.

The sensitivity analysis of the increasing online demand ratio revealed that the simulated results for the SFW and hybrid design strategies are not very different, even though there is a huge expected profit difference in the deterministic model. This means that the hybrid model within the deterministic model generates much better results than what firms might generate even in the presence of low demand variation. This result also implies that firms should take demand variation into account when they design their omnichannel distribution network strategies.

The results of the sensitivity analysis also show that the realized profit (the simulated results) declines rapidly for all channel design strategies even though the expected profit (the result of the deterministic model) decreases slightly as the coefficient of variation increases. The contribution of the dynamic assignment policy to the SFW strategy becomes more important as the variation increases; however, at a very high level of demand variation (coefficient of variation ≥ 0.4), the impact of the dynamic assignment on the total profit is diminished. A comparison of the results for the SFW and hybrid design strategies in the increased level of variation reveals that the total profits in simulation models are not very different, even though there seems to be a substantial profit difference in deterministic settings. This implies that when there is a high level of demand variation on the market, having inventory available at both warehouses and RSs becomes increasingly important. The channel design strategy (either the SFW or hybrid strategy), however, plays a minor role.

The developed approach is also used to investigate how the results react to simultaneous changes in both the coefficient of variation in demand and the desired CSL. The results indicate that the role of the dynamic assignment policy becomes more important when the coefficient of variation is high and a high level of CSL is desired. However, SFS strategies do not cope very well with demand variation at a high level of CSL. Interestingly, SFW strategies are more profitable than hybrid strategies in some cases due to the availability of more products at central repositories in the former, compared to the latter. At a high level of demand variation and a high desired CSL, the average distance between the customer and shipment points increases, and SFSs become more expensive.

The followed approach was applied to the Daskin dataset. The model could be applied to different geographical locations. An important limitation of the study concerns the lack of a sophisticated inventory model at warehouses. In the model, linearization was used to define the decreasing nature of the inventory holding costs per unit as the total volume increased. A more sophisticated model of the inventory holding costs could be defined. Of course, that would require nonlinear programming techniques, which were avoided in this study. Another limitation of the study is related to the cost parameters adopted from Millstein et al. (2022). Different cost structures could be used to investigate how the results change due to changes in the cost parameters.

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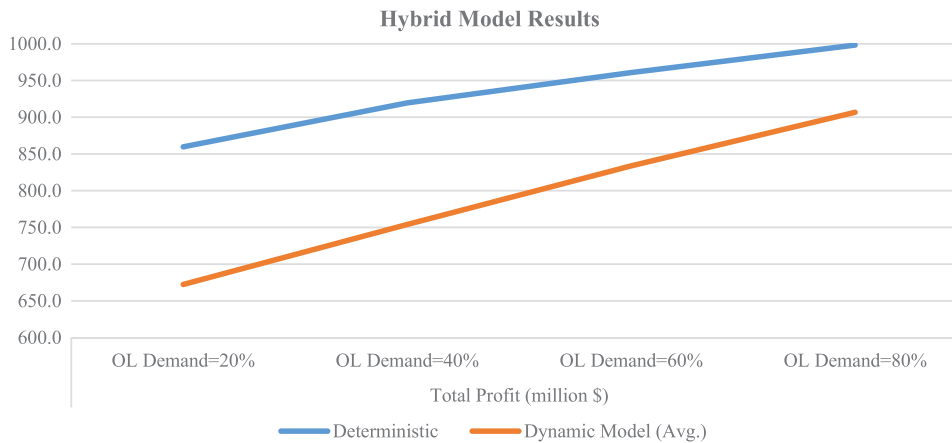
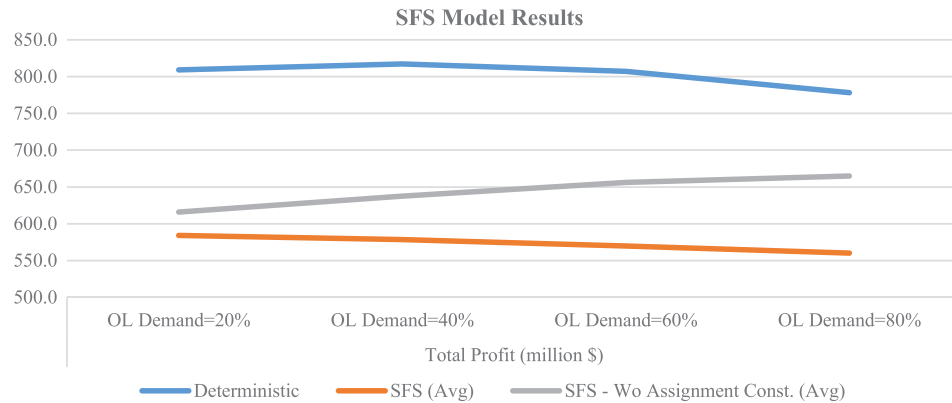
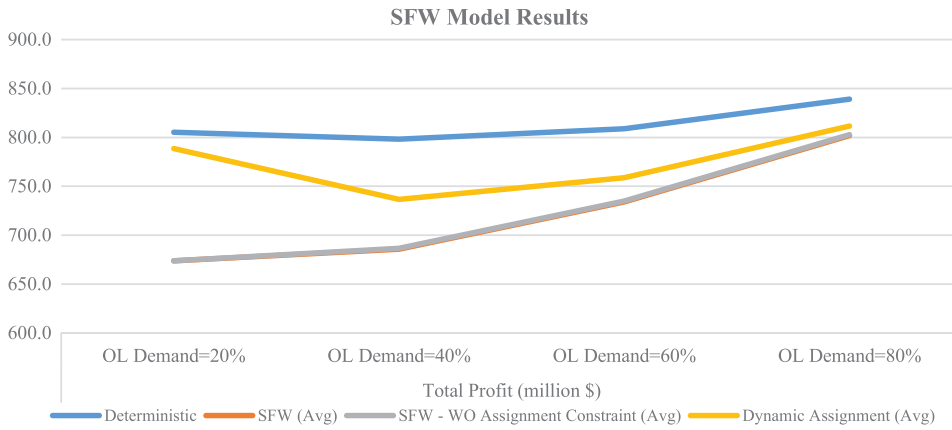
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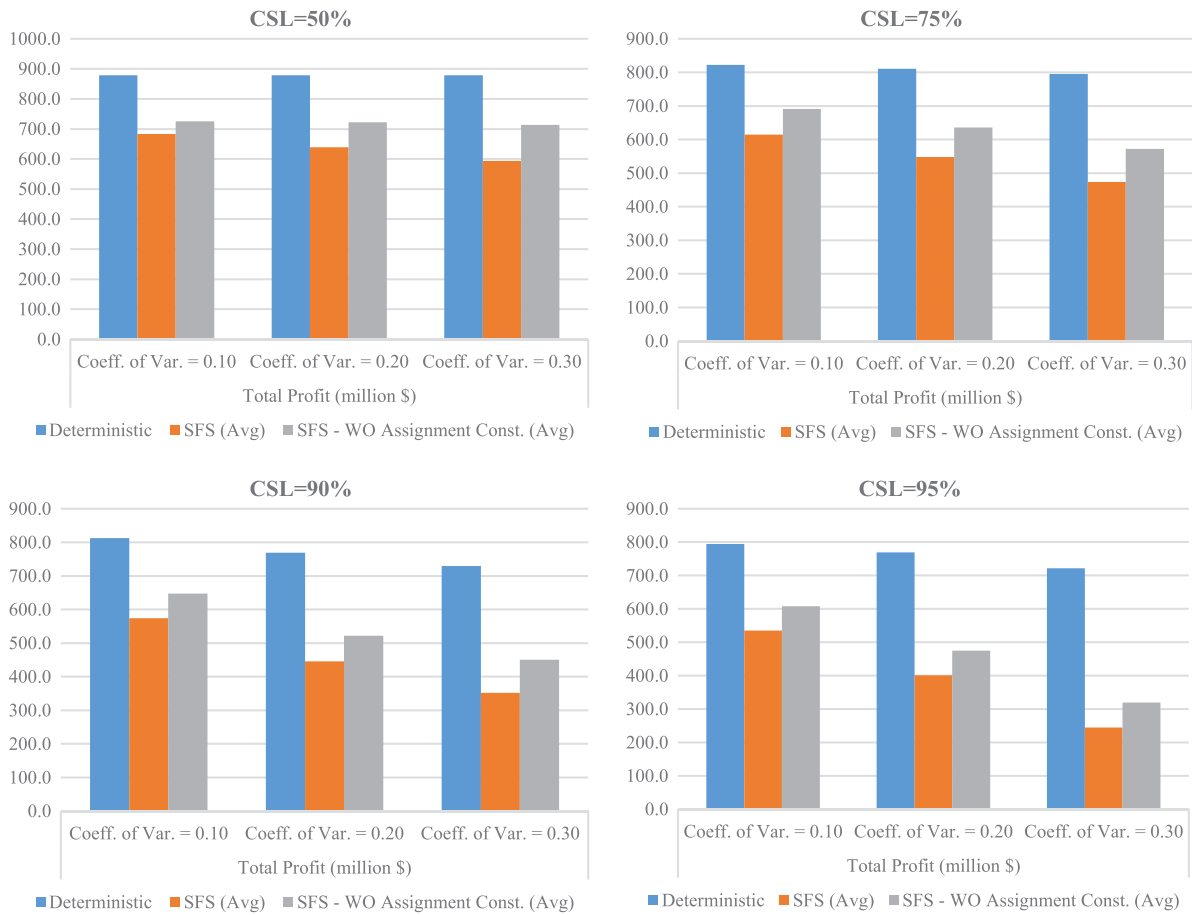
Appendix 1: Results for various channel design strategies as online demand ratio increases



Appendix 2: Results for various channel design strategies as the coefficient of variation of demand increases



SFS Model Results



Hybrid Model Results

