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FUZZY SYSTEMS AND THEIR MATHEMATICS

A novel measure of entropy and COPRAS based on Fermatean fuzzy sets and its analysis of early childhood feeding behaviors

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Abstract
This study aims to address early childhood nutritional behaviors from a sustainability perspective, evaluating the multidimensional factors that influence them within an integrated decision-making framework. Considering the decisive role of nutritional habits acquired in early childhood on an individual's lifelong health status and the sustainability of societal health systems, these behaviors need to be considered not only in their biological but also in their environmental, behavioral, and socioeconomic dimensions. In this context, a novel entropy measure based on Fermatean fuzzy sets has been developed for the field of early childhood nutrition, where uncertainty and subjective evaluations are intense, and a unique multi-criteria decision-making approach has been proposed by integrating it with the COPRAS method. To demonstrate the applicability of the proposed method, we evaluated sustainable nutritional behaviors for early childhood across four basic alternatives (seasonal feeding, natural feeding, adequate and balanced feeding, and economical feeding) and within a framework of numerous criteria that represent these alternatives. We converted linguistic evaluations based on expert opinions into Fermatean fuzzy numbers, criterion weights were objectively determined using the developed entropy approach, and alternatives were ranked using the COPRAS method. The findings indicate that the natural feeding approach has the highest priority in establishing sustainable eating behaviors in early childhood, followed by seasonal feeding, adequate and balanced nutrition, and economically viable nutritional alternatives. The results reveal that sustainability in early childhood is built not through cognitive awareness, but through repeatable, internalized behaviors in daily life. This study offers a holistic perspective based on behavioral sustainability to the early childhood nutrition literature. It contributes a new entropy measure and an applicable framework for multi-criteria decision-making to the Fermatean fuzzy sets literature.

Keywords Fermatean fuzzy set · Sustainability · Early childhood · Feeding behaviors · Decision-making · Entropy · COPRAS

1 Introduction

Early childhood is a critical stage in which the foundations for lifelong health and well-being are laid. The sustainability of society's health, as well as an individual's future physical and mental development, is significantly impacted by the nutritional habits developed during this time. A complex interplay of genetic, environmental, psychological, and cultural factors shapes nutritional practices. Parents and primary caregivers are important players in this process, influencing children's dietary experiences and surroundings. By modeling their own eating habits, tastes, and food preferences, parents not only provide nutritious conditions for their children's early food encounters but also influence their children's eating habits (Kral and Rauh 2010). According to Costa and Oliveira (2023), these behaviors are caused by a mix of environmental and hereditary variables. The foundation for creating long-term plans for a healthy future is an understanding of these relationships. This study's primary goal is to examine early childhood dietary behaviors from a sustainability perspective and to identify the multifaceted elements influencing these behaviors using an integrated decision-making framework.

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1.1 Research motivation

The primary motivation for this research is the lasting and profound impact of early childhood nutritional habits on an individual's future health. Nutritional habits developed throughout childhood are likely to continue into adulthood, according to Clark et al. (2007). As a result, eating poorly as a child has long-term consequences for one's health. The seriousness of the issue is further increased by its persistence as well as the rising incidence of diseases, including pediatric obesity and eating disorders, which are currently a global health concern. The rise in childhood obesity is concerning because obese children not only have risk factors for long-term conditions like low high-density lipoprotein cholesterol, hypertension, hyperlipidemia, and hyperinsulinemia, but they can also carry these risk factors into adulthood, which can have detrimental long-term health effects (Kral and Rauh 2010). These problems are not only an individual tragedy but also a societal burden that places unsustainable pressure on healthcare systems.

The second key motivating factor is the complexity of the family environment in which these behaviors are shaped. Nutrition is far more than a simple biological act; it is an arena where countless factors intertwine, including parents' own eating habits, nutritional beliefs, cultural norms, and the child's temperament. Ningning and Wenguang (2023) emphasize this central role, stating that the family parenting environment, a crucial area for children's growth and development, plays a significant role in shaping eating behaviors in preschool children. Parenting styles are associated with various nutritional outcomes; for example, a study by Leuba et al. (2022) found that negative parenting styles were consistently linked to problematic eating behaviors. Understanding this complex and multifaceted structure and showing how healthy eating habits can be sustainably promoted constitutes the main driving force of this study.

1.2 Methodological motivation: the secant-based entropy

While various entropy measures have been proposed for FFSs, many of them rely on linear or logarithmic functions that may exhibit limited sensitivity in distinguishing highly uncertain or near-equilibrium states, where the membership and non-membership degrees are close to each other. To address this, we propose a new entropy measure based on a secant function. The secant function provides a nonlinear response, which is particularly advantageous in the proximity of $\zeta^3_{\mathscr{F}}(a) \approx \eta^3_{\mathscr{F}}(a)$, a region where many existing entropy measures tend to saturate. This feature allows the proposed measure to capture subtle variations in the uncertainty of expert evaluations more effectively. A comparative analysis with existing entropy measures is provided in Sect. 3 to demonstrate its superior discriminatory power.

1.3 Necessity

While the current literature in the field of early childhood nutrition is rich, it exhibits a significantly fragmented structure. Studies often focus on a single aspect of parent–child interaction, neglecting the dynamic and bidirectional nature of this relationship. The intricacy of cause-and-effect interactions is emphasized by Costa and Oliveira (2023), who note the uncertainty about whether parental feeding practices are causes or predictors of children's eating habits. For example, according to the findings of Gebru et al. (2021), children who were forced to eat by their caregivers became more selective/irritable about food. This raises the question of whether parental pressure leads to selective eating or whether selective eating drives parents to exert pressure. Holistic and integrated models are urgently needed to understand these cyclical interactions. The significant gaps in the literature further reinforce this need. The role of fathers has been systematically neglected. Litchford et al. (2020) note that while evidence is growing regarding the influence of fathers on children's feeding habits, research in this area remains insufficient. Similarly, the impact of socioeconomic stressors, such as food insecurity, on parental feeding behaviors has not been adequately studied (Orr et al. 2020).

1.4 Originality

The fundamental originality of this study lies in its integration of early childhood nutritional behaviors under two innovative lenses: "sustainability" and "integrated decision-making." The concept of sustainability is broadened here to include not only environmental factors but also an individual's capacity to maintain healthy eating habits throughout life and the intergenerational transmission of these behaviors. As Coulthard et al. (2004) point out, understanding the mechanisms of intergenerational transmission of these behaviors is critical. This approach integrates micro-level family dynamics with macro-level ecological and environmental systems. For example, the study by Hristov et al. (2020), which emphasizes the vital role of pollinators in agricultural production, demonstrates how ecosystem health directly affects the variety and quality of food reaching families' tables. This reveals that nutritional sustainability cannot be considered separately from ecological sustainability. Another unique aspect of the study is its effort to synthesize numerous different theoretical and empirical findings into a single model. This model integrates the nuances between authoritarian and non-authoritarian feeding styles of parents (Carnell et al. 2014), the developmental stages of children's feeding skills (Sdravou et al. 2023), the process

by which caregivers' beliefs and intentions regarding feeding translate into behavior (Oudat et al. 2025), and feeding practices in different cultural contexts, Yuan et al. (2021). This synthesis encompasses a broad historical and theoretical spectrum, ranging from Baldwin's (1945) foundational work on appetite, pickiness, and table manners to modern methodological explorations such as observational coding by Pesch and Lumeng (2017). This offers a holistic and dynamic perspective on the subject, moving beyond fragmented approaches.

1.5 Contribution

This study has the potential to make significant contributions in four key areas of early childhood nutrition: theory, practice, methodology, and policy. Theoretically, it will combine findings from different disciplines to provide an integrated conceptual framework explaining the sustainability of early childhood feeding behaviors. This framework will position parental behaviors not as simple factors, but as the dynamic outcome of a complex decision-making process.

Practically, the study will generate directly applicable information and strategies for parents, caregivers, and health professionals. According to Clark et al. (2007), parents who are concerned about their children's weight need guidance and encouragement to change their feeding habits. Our findings can help inform the development of interventions for problematic eating behaviors, such as picky eating (Lewinsohn et al. 2005).

Methodologically, this study will respond to the call for further development of tools, as highlighted by Lauzon-Guillain et al. (2012). Specifically, it will provide a roadmap for developing new measurement tools that can better capture the two-way nature of parent–child interaction and cultural differences. Ultimately, this study will provide fundamental data for the development of evidence-based public health policies that support children in forming healthy and sustainable eating habits. These policies will aim to build a sustainable future by not only educating families but also providing them with the necessary resources and supportive environment to make healthy choices.

2 Literature review

Sustainable nutrition is a multifaceted and integrated paradigm that goes beyond supporting the healthy growth and development of individuals, while also considering environmental integrity and economic equity. According to the definition by the Food and Agriculture Organization (FAO) and the World Health Organization (WHO), sustainable diets guarantee food security and nutritional sufficiency for present and future generations with a low environmental impact, while simultaneously considering food diversity, environmental sustainability, and economic accessibility (FAO and WHO 2019). This inclusive approach is critically important for shaping both individual health outcomes and long-term impacts on societal and ecological systems, particularly during early childhood (0–8 years), when the foundations of lifelong behavioral patterns are laid (UNICEF 2021). Early childhood educational settings (home, school, and community) stand out as primary contexts where children's food preferences and eating behaviors are shaped through social learning; the integration of sustainable dietary models into this tripartite structure is increasingly emphasized in the literature (Mazzocchi et al. 2022). However, the factors constituting sustainable nutrition (seasonality, naturalness, adequacy, economics) often involve uncertainties that cannot be defined with precise boundaries, are interdependent, and vary across contexts. This multi-criteria and fuzzy structure demonstrates that flexible decision-support systems, such as fuzzy logic developed by Zadeh (1965), are an ideal methodological tool for modeling the optimal balance of these factors in the home-school-community triangle and for providing policymakers with a quantitative framework (Sezgin et al. 2023).

Seasonal eating refers to consuming foods in accordance with their natural production cycles, primarily sourced from local and regional sources. The impact of this approach on sustainability primarily relates to environmental benefits, such as reduced greenhouse gas emissions (mainly from long-distance transportation and greenhouse farming) and a smaller water footprint. Furthermore, produce harvested in season and at its ripest stage generally has a higher nutritional value (e.g., antioxidant and vitamin content) and a more robust flavor profile. Studies in Europe show that promoting seasonal and regional produce within sustainable eating patterns contributes to societal sustainability by increasing dietary diversity and supporting local agricultural economies (Vargas et al. 2021). In early childhood, planning school menus with seasonal produce or having families shop for seasonal produce at markets allows children to establish a concrete connection with natural cycles and expand their nutritional perceptions by introducing them to seasonal tastes. The fact that seasonality is a gradual concept (e.g., whether a product is "in season") and varies by geography makes it meaningful to integrate this factor into sustainable nutrition decisions in a flexible way using fuzzy logic models.

Natural nutrition centers on the consumption of minimally processed, unrefined foods free of artificial additives. This approach is directly linked to both the health and environmental dimensions of sustainable diets. Sustainable nutrition models aim to reduce the ecological impacts of animal production and the highly processed food industry, such as land use, water consumption, and pollution, by promoting the consumption of plant-based and natural foods (whole grains, legumes, fruits and vegetables, nuts) (Eat-Lancet

2019). Models like EAT-Lancet's "Planetary Health Diet" have translated these principles into concrete recommendations. Feeding children with natural and minimally processed foods not only optimizes macronutrient and micronutrient intake but also allows children to have positive experiences with the taste, texture, and smell of real foods (Mazzocchi et al. 2021). Furthermore, the association between ultra-processed food consumption and childhood obesity is among the findings supporting the importance of natural nutrition (Chang et al. 2021). The fact that the concept of "naturalness" is a relative and continuous quality (from "less processed" to "more processed") makes it highly suitable for modeling with fuzzy sets.

A sustainable nutritional model must inevitably guarantee nutritional sustainability; that is, it must provide all macro (carbohydrates, proteins, fats) and micro (vitamins, minerals) nutrients adequately and in a balanced way, appropriate to the child's age, gender, and developmental level. This concept is the cornerstone of the individual health aspect of sustainable nutrition. The literature shows that sustainable and plant-based nutritional models, such as the Mediterranean diet, can help reduce the risk of obesity, type 2 diabetes, and cardiovascular disease in children and adolescents while ensuring nutritional adequacy (Teixeira et al. 2022). Sustainable nutrition aims not only to achieve adequate and balanced nutrition as a health goal, but also to do so by minimizing food waste, prioritizing nutrient-dense foods, and promoting dietary diversity. At this point, educators' and parents' nutritional literacy plays a critical role. The fact that concepts such as "adequacy" and "balance" are based not on absolutes but on individual needs and tolerable ranges underscores the usefulness of fuzzy logic's flexible limits in evaluating these factors.

Affordable nutrition is central to the societal dimension of sustainable diets. This factor encompasses not only the affordability of nutritious foods but also households' social and physical access to them. Affordable accessibility is key to ensuring that sustainable and healthy nutrition becomes the norm, not a privilege, for broad segments of society. Global studies show that sustainable diets, such as the EAT-Lancet reference diet, may be more expensive than many households in low- and middle-income countries can afford. However, costs can be reduced through optimization and local substitutions (Hirvonen et al. 2020). Furthermore, the long-term impact of healthy and sustainable diets on reducing healthcare costs should be considered. In early childhood, school meal programs and nutritional education and support programs for families stand out as effective policies in reducing nutritional deficiencies caused by economic inequalities (WHO 2023). The intersection and uncertainty between the quantitative (income, price) and qualitative (accessibility, acceptability) components of the economic factor clearly demonstrate how a fuzzy-logic-based model can help manage this complexity.

Multi-Criteria Decision Making (MCDM) is a systematic approach to selecting, ranking, or making decisions among alternatives that considers multiple criteria simultaneously. These methods integrate both quantitative and qualitative criteria, enabling more realistic, transparent, and structured decisions. MCDM is widely used when single-criterion evaluations are insufficient, as real-world decision problems are often multidimensional. Decision makers must address issues across numerous domains in a methodical, accurate, and dependable manner, aligned with their preferences. Since every situation calls for a different choice, MCDM is especially helpful in this field. MCDM uses the solutions chosen by decision-makers to address complex issues. In particular, MCDM helps DMs understand the problem at hand by breaking it down into manageable chunks. These advantages have led to the widespread application of MCDM in a variety of disciplines, including management, economics, medicine, the environment, education, and energy. The primary goals of MCDM are to increase decision quality, efficiency, rationality, and explicitness. The main reasons why MCDM methods are needed can be explained as follows: The necessity of evaluating multiple conflicting criteria simultaneously, facilitating rational and consistent choice formation by human decision-makers, structuring processes when working with complex options, ensuring that decision outcomes are explainable and repeatable, and being effective, especially in problems involving uncertainty, subjectivity, and conflicting objectives (Nayli et al. 2021)

The MCDM field began to institutionalize in the 1970 s and has grown steadily ever since. Methodologies such as Analytical Frameworks, AHP, and ELECTRE became popular during those years. According to recent literature reviews, over 10,000 studies on MCDM methodology have been published between 1974 and 2024, and the field continues to grow rapidly (Kaya 2024).

The MCDM analysis process typically involves the following steps (Nayli et al. 2021):

1. Defining the problem
2. Determining the decision criteria
3. Identifying alternatives
4. Creating criterion weights
5. Selecting the evaluation method
6. Calculating and ranking alternatives
7. Interpreting the results.

This structure both makes the decision-making process transparent and ensures repeatable analysis. MCDM methods are powerful tools that yield analytical, repeatable solutions in complex, multi-dimensional decision-making processes. Both classical methods and new approaches are constantly being developed in the literature, applied across various disciplines, and the number of methodological studies is rapidly increasing (Kaya 2024).

Classical MCDM methods (AHP, TOPSIS, VIKOR, WASPAS, etc.) generally work with crisp data. However, in real DM problems, decision-makers use linguistic expressions (such as high, medium, low), preferences can be ambiguous, uncertain, or contradictory, and we cannot express expert opinions with full membership. Therefore, in the literature, MCDM methods have been combined with different extensions of fuzzy set (FS) theory (Zadeh 1965). In fuzzy logic, an element belonging to a set is expressed by a single degree of membership. Thus, criterion weights are expressed as fuzzy numbers, and linguistic evaluations are included in the model. Fuzzy decision techniques are relatively simple computationally and are the most widely used fuzzy structure in the literature.

A generalization of the FS, the Intuitionistic Fuzzy Set (IFS) (Atanassov 1986) defines membership, non-membership, and hesitation degree for each element. The decision-maker's states, such as "Yes," "No," and "I am not sure," can be modeled independently by the IFS. As a result, the IFS is better suited for collaborative decision-making and can explicitly model uncertainty, which aligns more closely with human thought processes. A generalization of the IFS is the Pythagorean Fuzzy Set (PFS) (Yager 2013, 2014). As long as the total of the squares of the membership and non-membership degrees is less than or equal to 1, it gives the decision-maker more freedom to express themselves. PFSs more accurately reflect expert judgments and provide a more flexible portrayal of uncertainty. A generalization of the IFS and PFS is the Fermatean Fuzzy Set (FFS). As long as the sum of the squares of the membership and non-membership degrees is less than 1, FFS (Senapati and Yager 2019a, b, 2020) can be applied with ease to choice issues with extremely high uncertainty. To put it another way, FFS is used to represent more complicated, realistic decision-making settings and enables the decision-maker to articulate both membership and non-membership degrees with high confidence.

IFS, PFS, and FFS, new MCDM techniques based on these sets, have recently been increasingly contributed to the literature. In Kirişci (2019), Fibonacci statistical convergence on an IF-normed space was improved. In Tokede (2025), a study on the application of IFS in a social life-cycle impact assessment of a public building project was conducted. This study proposes combining IFS with multi-qualitative decision-making to transform qualitative data into quantitative estimates of the building project's social impacts. In order to lower production costs and energy consumption in the machining of lightweight magnesium alloys, Sivam et al. (2025) evaluated and optimized four crucial machining parameters of ZE41 magnesium alloy at three levels using IFS-TOPSIS. With the growing emphasis on sustainable production, this need has become increasingly important. Zareef and Riaz (2025) proposed novel agglutination operators called the IF-credibility fairly weighted average operator and the IF-credibility fairly ordered weighted average operator. Additionally, the CoCoSo method is employed to rank the options, and the CRITIC method is used to determine the criteria weights. The group-based generalized PFSS and the generalized PFSS are defined in Kirişci (2024a). To measure the degree of smartphone addiction among teenagers, Ertemel et al. (2023) propose an integrated fuzzy MCDM approach that combines the CRITIC and TOPSIS methodologies based on PFSs. The theoretical underpinnings for discrete, linear, and zigzag PF uncertainty distributions are established in the study by Habib and Akram (2024), along with their operational rules and mathematical formulation. To assist businesses in selecting the optimal solution, the article in Mahapatra et al. (2025) proposes a comprehensive, adaptable MCGDM technique based on the PFS. Jeevaraj (2021) defined interval-valued Fermatean fuzzy sets (IVFFS). Kirişci (2023) expanded the four types of correlation coefficients for Fermatean hesitant FSs to the correlation and weighted correlation coefficients for interval-valued Fermatean hesitant FSs. To overcome the problem that the cardinalities of Fermatean hesitant fuzzy elements and interval-valued Fermatean hesitant fuzzy elements differ, we introduce least common multiple expansion techniques during processing. Lukman et al.'s research (Luqman et al. 2024) attempts to develop a novel approach for multi-objective optimization within the framework of the hesitant FF-methodology, incorporating both ratio analysis and complete multiplicative form techniques, in order to effectively address MCGDM challenges arising from uncertain and ambiguous data on the criterion set's weight values. An integrated MCDM approach based on interval-valued FFSs, including entropy, PIPRECIA, and MARCOS, is presented in Kirişci (2024b). To address various issues in sustainable supply chain management, Farid et al. (2023) proposed the FF-topology and used the FF-CODAS approach. An interval-valued FF-linguistic kernel principal component analysis model is constructed in Kirişci and Simsek (2023) in order to decrease dimensionality for nonlinear features. An integrated MCDM approach based on an interval-valued Fermatean fuzzy set that integrates the CRITIC, PIPRECIA, and MARCOS methodologies is presented in the work by Kirişci (2025a). The purpose of this study is to pinpoint the main obstacles keeping businesses from implementing IoT. To classify and rank the risks related to decentralized finance, Kirişci (2025b) used MCDM approaches, including IVFF-based DEMATEL, AHP, and TOPSIS. An integrated strategy based on the WASPAS method is suggested in Kirişci et al. (2025a) to successfully handle DM problems using interval-valued FFSs. To promote sustainability, the study in Kirişci et al. (2025b) proposes an integrated MCDM approach that combines SWARA and WASPAS using an interval-valued FFS. This approach successfully models and ranks the haz-

ards related to medical waste disposal. The authors of Kuzu et al. (2025) seek to address choice issues with interval-valued FFSs; the present study proposes a strategy based on the AHP.

The literature has examined several early childhood decision-making and decision-support systems. By constructing a fuzzy decision support system with fuzzy rules and a membership degree function to overcome hurdles and improve the efficiency of decision-making, Li (2025) sought to enhance the conduct of young children. In order to assess the regulatory and key elements influencing early childhood education, Kukreja et al. (2023) employed a hybrid fuzzy AHP-fuzzy TOPSIS to prioritize the critical factors found. Hou (2025) introduced a preschool instructional assessment methodology that uses a fuzzy decision-support system to enhance evaluations depending on predetermined factors, hence improving kids' learning experiences. A fuzzy ANP and a TOPSIS framework were used in Zhang et al. (2025) to investigate the effects of playful English-learning settings on self-regulated learning and English competence in early childhood education. Four algorithms and the process principles of the recently defined generalized PFSS VIKOR approaches are provided in the publication by Kirişci et al. (2022). The behavioral and cognitive development of young children under the COVID-19 quarantine was investigated as an application of these methods.

3 Preliminaries

Definition 3.1 (Senapati and Yager 2020) Let E be the universal set. The FFS is defined as the set $\mathscr{F} = \{(x, \zeta_{\mathscr{F}}(x), \eta_{\mathscr{F}}(x)) : x \in E\}$, where with $0 \leq \zeta^3_{\mathscr{F}} + \eta^3_{\mathscr{F}} \leq 1$ and $\zeta_{\mathscr{F}}, \eta_{\mathscr{F}} \in [0, 1]$. The hesitation degree has been shown with $h_{\mathscr{F}} = (1 - \zeta^3_{\mathscr{F}} + \zeta^3_{\mathscr{F}})^{1/3}$.

Consider the three FFSs $\mathscr{F} = (\zeta_{\mathscr{F}}, \eta_{\mathscr{F}})$, $\mathscr{F}_1 = (\zeta_{\mathscr{F}}, \eta_{\mathscr{F}})$, $\mathscr{F}_2 = (\zeta_{\mathscr{F}}, \eta_{\mathscr{F}})$. Then,

(i) $\mathscr{F}_1 \cap \mathscr{F}_2 = \big(\min(\zeta_{\mathscr{F}_1}, \zeta_{\mathscr{F}_2}), \max(\eta_{\mathscr{F}_1}, \eta_{\mathscr{F}_2})\big)$,
(ii) $\mathscr{F}_1 \cup \mathscr{F}_2 = \big(\max(\zeta_{\mathscr{F}_1}, \zeta_{\mathscr{F}_2}), \min(\eta_{\mathscr{F}_1}, \eta_{\mathscr{F}_2})\big)$,
(iii) $\mathscr{F}^c = (\eta_{\mathscr{F}}, \zeta_{\mathscr{F}})$,
(v) $\mathscr{F}_1 \otimes \mathscr{F}_2 = \left(\zeta_{\mathscr{F}_1}\zeta_{\mathscr{F}_2}, \sqrt[3]{\eta^3_{\mathscr{F}_1} + \eta^3_{\mathscr{F}_2} - \eta^3_{\mathscr{F}_1}\eta^3_{\mathscr{F}_2}}\right)$,
(v) $\mathscr{F}_1 \otimes \mathscr{F}_2 = \left(\zeta_{\mathscr{F}_1}\zeta_{\mathscr{F}_{21}}, \sqrt[3]{\eta^3_{\mathscr{F}_1} + \eta^3_{\mathscr{F}_2} - \eta^3_{\mathscr{F}_1}\eta^3_{\mathscr{F}_2}}\right)$,
(v) $\alpha\mathscr{F} = \left(\sqrt[3]{1 - (1 - \zeta^3_{\mathscr{F}})^{\alpha}}, (\eta^3_{\mathscr{F}})^{\alpha}\right)$,
(vi) $\mathscr{F}^{\alpha} = \left((\zeta^3_{\mathscr{F}})^{\alpha}, \sqrt[3]{1 - (1 - \eta^3_{\mathscr{F}})^{\alpha}}, \right)$.

Definition 3.2 (Rani and Mishra 2022) For the FFS $\mathscr{F}$,

$$SC(\mathscr{F}) = \frac{1}{2}\left(\zeta^3_{\mathscr{F}}(a) - \eta^3_{\mathscr{F}}(a)\right) \tag{1}$$

$$AC(\mathscr{F}) = \frac{1}{2}\left(\zeta^3_{\mathscr{F}}(a) + \eta^3_{\mathscr{F}}(a)\right) \tag{2}$$

$$\overline{SC}(\mathscr{F}) = \frac{1}{2}\left(SC(\mathscr{F}) + 1\right) \tag{3}$$

are called score, accuracy, and normalized score functions, respectively, where $SC(\mathscr{F}) \in [-1, 1]$, $AC(\mathscr{F}) \in [0, 1]$, and $\overline{SC}(\mathscr{F}) \in [0, 1]$.

Definition 3.3 For $i = 1, 2, \ldots, n$, consider a set of FFSs $\mathscr{F}_i = (\zeta_{\mathscr{F}}, \eta_{\mathscr{F}})$. The equation

$$FFWA(\mathscr{F}_1, \ldots, \mathscr{F}_n) = \left(\sum_{i=1}^{n} \omega_i \zeta_{\mathscr{F}_i}, \sum_{i=1}^{n} \omega_i \eta_{\mathscr{F}_i}\right) \tag{4}$$

is called the FF-weighted average (FFWA) operator, and the equation

$$FFWG(\mathscr{F}_1, \ldots, \mathscr{F}_n) = \left(\prod_{i=1}^{n} \zeta^{\omega_i}_{\mathscr{F}_i}, \prod_{i=1}^{n} \eta^{\omega_i}_{\mathscr{F}_i}\right) \tag{5}$$

is called the FF-weighted geometric (FFWG) operator, where ω_i is a weight vector of $\mathscr{F}_i$ with $\sum_{i=1}^{n} \omega_i = 1$.

4 New entropy measure

A new entropy metric is provided in this subsection. To produce more nonlinear responses to entropy, this innovation also uses a flexible secant function. These solutions align well with the current trend of developing tools that address nonlinearity, such as MCDM.

Definition 4.1 The mapping $\mathscr{E} : FFS(E) \to [0, 1]$ is called a Fermatean fuzzy entropy if the following conditions hold:

E1. $0 \leq \mathscr{E}(\mathscr{F}) \leq 1$.
E2. $\mathscr{E}(\mathscr{F}) = 0$ iff $\mathscr{F}$ is a crisp set.
E3. $\mathscr{E}(\mathscr{F}) = 1$ iff $\zeta_{\mathscr{F}}(a) = \eta_{\mathscr{F}}(a)$.
E4. $\mathscr{E}(\mathscr{F}) \leq \mathscr{E}(\mathscr{G})$ iff
- $\mathscr{F} \subseteq \mathscr{G}$, if
- $\mathscr{G} \subseteq \mathscr{F}$, if
E5. $\mathscr{E}(\mathscr{F}^c) = \mathscr{E}(\mathscr{F})$ for each $\mathscr{E}(\mathscr{F}) \in FFS(E)$.

Definition 4.2 Consider the FFS $\mathscr{F} = \{(a, \zeta_{\mathscr{F}}(a), \eta_{\mathscr{F}}(a)) : a \in E\}$. The Fermatean fuzzy entropy of $\mathscr{F}$ is defined as:

$$\mathscr{E}(\mathscr{F}) = \frac{1}{n}\sum_{i=1}^{n}\left[\sec\left(\frac{\pi}{3} - \frac{|\zeta^3_{\mathscr{F}}(a) - \eta^3_{\mathscr{F}}(a)|}{3}\pi\right) - 1\right]. \tag{6}$$

Theorem 4.3 *$\mathscr{E}(\mathscr{F})$ is a FF-entropy.*

Proof We will show $0 \leq \mathscr{E}(\mathscr{F}) \leq 1$. It is known to be $0 \leq \zeta^3_{\mathscr{F}}(a), \eta^3_{\mathscr{F}}(a) \leq 1$ and $0 \leq \zeta^3_{\mathscr{F}}(a) + \eta^3_{\mathscr{F}}(a) \leq 1$. Further, $\frac{\pi}{3}|\zeta^3_{\mathscr{F}}(a) - \eta^3_{\mathscr{F}}(a)| \leq \frac{\pi}{3}$. Therefore, $0 \leq \mathscr{E}(\mathscr{F}) \leq 1$.

If $\mathscr{E}(\mathscr{F}) = 0$, then $\frac{\pi}{3}|\zeta^3_{\mathscr{F}}(a) - \eta^3_{\mathscr{F}}(a)| = \frac{\pi}{3}$. Thus, $\zeta_{\mathscr{F}}(a) = 0$, $\eta_{\mathscr{F}}(a) = 1$, $h_{\mathscr{F}}(a) = 0$ or $\zeta_{\mathscr{F}}(a) = 1$, $\eta_{\mathscr{F}}(a) = 0$, $h_{\mathscr{F}}(a) = 0$. This shows us that $\mathscr{F}$ is a crisp set. Conversely, if $\mathscr{F}$ is a crisp set, $\mathscr{E}(\mathscr{F}) = 0$.

If $\mathscr{E}(\mathscr{F}) = 1$, then $\frac{\pi}{3}|\zeta^3_{\mathscr{F}}(a) - \eta^3_{\mathscr{F}}(a)| = 0$. Hence, $\zeta_{\mathscr{F}}(a) - \eta_{\mathscr{F}}(a) = 0, \Rightarrow \zeta_{\mathscr{F}}(a) = \eta_{\mathscr{F}}(a)$. Conversely, if $\zeta_{\mathscr{F}}(a) = \eta_{\mathscr{F}}(a)$, $\mathscr{E}(\mathscr{F}) = 1$.

Write the function $U(x, y) = \sec\left(\frac{\pi}{3} - \frac{|x^3 - y^3|}{3}\pi\right) - 1$. To perform this operation, two situations will be taken into account: $x \leq y$ and $x \geq y$.

For $x \leq y$, $V(x, y) = \sec\left(\frac{\pi}{3} - \frac{|y^3 - x^3|}{3}\pi\right) - 1$. It must be proven that $V(x, y)$ increases with x and decreases with y. The partial derivatives for x and y are as follows:

$$\frac{\partial V(x, y)}{\partial x} = \pi x^2[\sec(V).\tan(V)] \geq 0,$$
$$\frac{\partial V(x, y)}{\partial y} = -\pi y^2[\sec(V).\tan(V)] \leq 0.$$

From this results, the conditions $\zeta_{\mathscr{G}}(a) \leq \eta_{\mathscr{G}}(a)$ and $\zeta_{\mathscr{F}}(a) \leq \zeta_{\mathscr{G}}(a)$, $\eta_{\mathscr{F}}(a) \geq \eta_{\mathscr{G}}(a)$ hold. Hence, $U(\zeta_{\mathscr{F}}(a), \eta_{\mathscr{F}}(a)) \leq U(\zeta_{\mathscr{G}}(a), \eta_{\mathscr{G}}(a))$.

For $x \geq y$, $Z(x, y) = \sec\left(\frac{\pi}{3} - \frac{|x^3 - y^3|}{3}\pi\right) - 1$. It must be proven that $V(x, y)$ decreases with x and increases with y. Then,

$$\frac{\partial Z(x, y)}{\partial x} = -\pi x^2[\sec(Z).\tan(Z)] \leq 0,$$
$$\frac{\partial Z(x, y)}{\partial y} = \pi y^2[\sec(Z).\tan(Z)] \geq 0.$$

That is, the conditions $\zeta_{\mathscr{G}}(a) \geq \eta_{\mathscr{G}}(a)$ and $\zeta_{\mathscr{F}}(a) \geq \zeta_{\mathscr{G}}(a)$, $\eta_{\mathscr{F}}(a) \leq \eta_{\mathscr{G}}(a)$ hold. Hence, $U(\zeta_{\mathscr{F}}(a), \eta_{\mathscr{F}}(a)) \leq U(\zeta_{\mathscr{G}}(a), \eta_{\mathscr{G}}(a))$. Therefore, $\mathscr{F} \leq \mathscr{G} \Rightarrow \mathscr{E}(\mathscr{F})\ leq \mathscr{E}(\mathscr{G})$.

Finally,

$$\begin{aligned}\mathscr{E}(\mathscr{F}) &= \frac{1}{n}\sum_{i=1}^{n}\left[\sec\left(\frac{\pi}{3} - \frac{|\zeta^3_{\mathscr{F}}(a) - \eta^3_{\mathscr{F}}(a)|}{3}\pi\right) - 1\right] \\ &= \frac{1}{n}\sum_{i=1}^{n}\left[\sec\left(\frac{\pi}{3} - \frac{|\eta^3_{\mathscr{F}}(a) - \zeta^3_{\mathscr{F}}(a)|}{3}\pi\right) - 1\right] \\ &= \mathscr{E}(\mathscr{F}^c).\end{aligned}$$

□

Corollary 4.4 *The proposed entropy measure $\mathscr{E}(\mathscr{F})$ provides a more sensitive response to changes in uncertainty compared to several existing measures, particularly when the difference between the cubes of membership and non-membership degrees is small. This is due to the nonlinear nature of the secant function, which avoids saturation near the equilibrium point $\zeta^3_{\mathscr{F}}(a) = \eta^3_{\mathscr{F}}(a)$.*

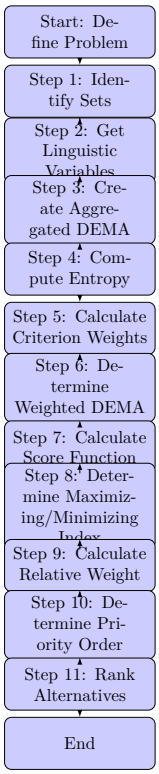


Fig. 1 Flowchart of the proposed integrated MCDM framework

Example 4.5 (Comparative analysis) To illustrate the sensitivity and discriminatory power of the proposed entropy measure, we compare it with a benchmark entropy measure commonly used in the literature for intuitionistic and Fermatean fuzzy sets. The benchmark entropy measure, denoted as $\mathscr{E}_b(\mathscr{F})$, is defined as follows (Szmidt and Kacprzyk 2001; Wang et al. 2018):

$$\mathscr{E}_b(\mathscr{F}) = \frac{1}{n}\sum_{i=1}^{n}\left(1 - \left|\zeta^3_{\mathscr{F}}(a_i) - \eta^3_{\mathscr{F}}(a_i)\right|\right). \tag{7}$$

This measure is linear and yields a value of 1 when $\zeta_{\mathscr{F}}(a_i) = \eta_{\mathscr{F}}(a_i)$ for all i, and a value of 0 when the set is crisp.

Now, consider the following three Fermatean fuzzy sets, each representing a different level of uncertainty:

- $\mathscr{F}_1 = \{(x, 0.7, 0.7)\}$ (High uncertainty, near equilibrium)
- $\mathscr{F}_2 = \{(x, 0.6, 0.6)\}$ (Moderate uncertainty)
- $\mathscr{F}_3 = \{(x, 0.5, 0.5)\}$ (Maximum uncertainty, equilibrium point)

For these sets, the condition $\zeta^3_{\mathscr{F}}(a) + \eta^3_{\mathscr{F}}(a) \leq 1$ is satisfied (e.g., for $\mathscr{F}_1$, $0.7^3 + 0.7^3 = 0.686 \leq 1$). The computed

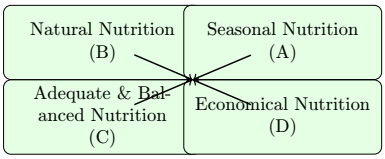


Fig. 2 Sustainable nutrition alternatives in early childhood

entropy values for both the proposed measure $\mathscr{E}(\mathscr{F})$ and the benchmark measure $\mathscr{E}_b(\mathscr{F})$ are shown in Table 1.

As clearly shown in Table 1, the proposed entropy measure $\mathscr{E}(\mathscr{F})$ yields distinct and decreasing values (0.987, 0.876, 0.500) as the uncertainty decreases (i.e., as the membership and non-membership degrees move away from the equilibrium point). In contrast, the benchmark measure $\mathscr{E}_b(\mathscr{F})$ produces a constant value of 1.000 for all three sets, failing to discriminate between them and remaining saturated at its maximum. This comparative analysis demonstrates that the proposed secant-based entropy measure possesses superior discriminatory power, particularly in high-uncertainty environments where small differences in expert evaluations are critical for decision-making.

5 Proposed method

By combining the cutting-edge FF-entropy and COPRAS tools (Fig. 4), this study suggests an integrated method for handling MCDM problems under FFSs. Using pseudo-representations, the fundamental ideas of FFSs and related MCDM approaches have been explained separately (Figs. 1, 2, 3).

Let the sets of alternatives and criteria be given as follows, respectively: $A = \{A_1, A_2, \ldots, A_m\}$ and $K = \{K_1, K_2, \ldots, K_n\}$. A senior member assembled a group of l experts $(U_1, U_2, \ldots, U_l)$.

LEVEL A: Obtain the decision matrices based on experts.
Step 1: Identify the alternative, criteria and experts sets:
Step 2: Get the linguistic variables of experts and convert linguistic variables to FF-numbers.
Step 3: Create the aggregated DEMA based on FF-numbers by Eq. 4 (or 5) with DEs weights.
LEVEL B: Find the weights.
Step 4: Compute the entropy on FFSs of the criterion by Eq. 6.
Step 5: Calculate the weight of the criterion by Eq. 8.
LEVEL C: COPRAS method.
Step 6: Determine the weighted DEMA by Eq. 9.
Step 7: Calculate the score function $\overline{SC}(r_{ij})$ by Eq. 3.
Step 8: Determine the maximizing $\overline{SC}(P_i)$ and minimizing $\overline{SC}(R_i)$ index by Eqs. 10, 11.
Step 9: Calculate the relative weight of each alternative by Eq. 12.

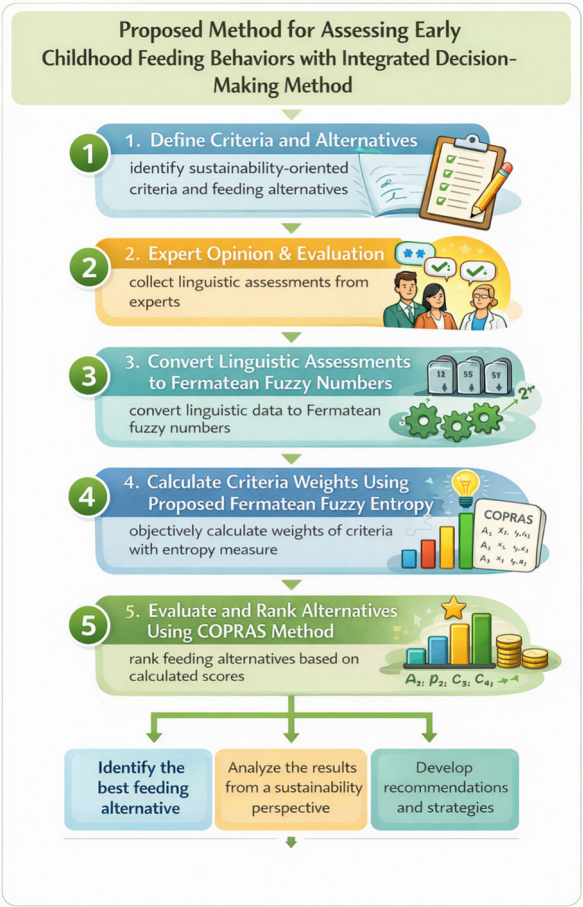


Fig. 3 Flowchart

Step 10: Determine the priority order by Eq. 13.
Step 11: If $P_{r_i} \geq P_{r_k}$, then rank the alternatives $Ai > Ak$.

$$\omega_j = \frac{1 - e_j}{\sum_{j=1}^{n}(1 - e_j)} \tag{8}$$

where e_j represented the entropy.

$$r_{ij} = \omega_j d_{ij} = \left(\sqrt[3]{1 - (1 - \zeta_{ij}^3)^{\omega_j}}, \eta_{ij}^{\omega_j}\right) \tag{9}$$

where $D = (d_{ij})_{m \times n}$ is a FF-decision matrix and $d_{ij} = [\zeta_{ij}, \eta_{ij}]$ is a FF-number representing the performance of value of alternative on the criterion.

$$\overline{SC}(P_i) = \frac{1}{|Y|} \sum_{j \in Y} \overline{SC}(r_{ij}), \tag{10}$$

$$\overline{SC}(R_i) = \frac{1}{|Z|} \sum_{j \in Z} \overline{SC}(r_{ij}), \tag{11}$$

Table 1 Comparison of entropy measures for different FFSs

Fermatean fuzzy set	$\zeta_{\mathscr{F}}, \eta_{\mathscr{F}}$	Proposed $\mathscr{E}(\mathscr{F})$	Benchmark $\mathscr{E}_b(\mathscr{F})$
$\mathscr{F}_1$	(0.7, 0.7)	0.987	1.000
$\mathscr{F}_2$	(0.6, 0.6)	0.876	1.000
$\mathscr{F}_3$	(0.5, 0.5)	0.500	1.000

Table 2 Scale values according to FFN

Linguistic terms	FFNs
Extremely high importance (EH)	(0.95, 0.15)
Very high importance (VH)	(0.85, 0.25)
High importance (H)	(0.75, 0.35)
Above average importance (AA)	(0.65, 0.45)
Equally importance (E)	(0.55, 0.55)
Below average importance (BA)	(0.45, 0.65)
Low importance (L)	(0.35, 0.75)
Very low importance (VL)	(0.25, 0.85)
Extremely low importance (EL)	(0.15, 0.95)

where Y and Z are the sets of benefit and non-benefit criteria, respectively.

$$Q_i = \overline{SC}(P_i) + \frac{\sum_{i=1}^{n} \overline{SC}(R_i)}{\overline{SC}(R_i) \sum_{i=1}^{n} \frac{1}{\overline{SC}(R_i)}}. \tag{12}$$

$$P_{r_i} = \frac{Q_i}{\max Q_i} \times 100. \tag{13}$$

6 Application

Table 2 shows linguistic terms and their corresponding FFNs.

Step 1: Alternatives are A-seasonal nutrition, B-natural nutrition, C- adequate and balanced nutrition, D- Economical nutrition. Sustainable nutrition alternatives in early childhood. Figure 4 illustrates four complementary approaches—natural nutrition, seasonal nutrition, adequate and balanced nutrition, and economic nutrition—integrated within a comprehensive sustainability framework. Each alternative represents a distinct dimension of sustainability and contributes differently to the development of long-term, behaviorally internalized eating habits in early childhood.

Natural Nutrition focuses on developing healthy, sustainable eating habits through daily experiences in early childhood. Key characteristics of this type of Nutrition include avoiding processed foods and products containing additives, preferring homemade meals, and choosing natural, minimally processed foods. In this sense, behavioral sustainability and long-term health sustainability demonstrate the sustainability dimension of this type of Nutrition.

Seasonal Nutrition emphasizes the consumption of foods produced and consumed during their natural growing seasons. Key characteristics of this diet include consuming fresh fruits and vegetables in season, preferring foods produced and stored correctly at that time, and reducing reliance on imported products. Environmental sustainability and ecological awareness represent the sustainability dimension of this type of diet.

Adequate and Balanced Nutrition ensures that children receive sufficient and nutritionally balanced meals that meet their developmental needs. Sufficient consumption of animal products, regular and timely meal times, and adequate intake of plant-based foods are considered fundamental characteristics. The sustainability aspect of this type of nutrition can be defined as physiological and health sustainability.

Economical Nutrition focuses on reducing waste and promoting the efficient use of food-related resources. Characteristics of economical eating include avoiding buying more food than needed, preventing food waste by finishing portions, and avoiding the use of disposable kitchenware. Economic sustainability and resource efficiency are aspects of sustainability.

Criteria: A1-Consuming fresh vegetables and fruits in season A2-Preferring foods that are bought in season and stored under appropriate conditions (frozen, dried, canned) A3-Not preferring imported products

B1-Not preferring packaged foods containing preservatives B2-Do not prefer processed foods (fast food, etc.) B3-Preferring home-cooked meals

C1-Consuming enough animal foods C2-Eating enough meals on time C3-Consuming enough plant foods

D1-Not shopping for more food than needed D2-Not leaving portions unfinished D3-Not preferring single-use kitchenware (plates, spoons, forks, etc.)

D_1, D_2, and D_3 are non-benefit criteria, and the others are benefit criteria.

Two of the experts whose evaluations were taken advantage of are faculty members of the Preschool Teaching Department, two are administrators in preschool education institutions, and one is a preschool teacher. The faculty members have 17 and 20 years of experience. The school principals have 18 and 23 years of experience, and the

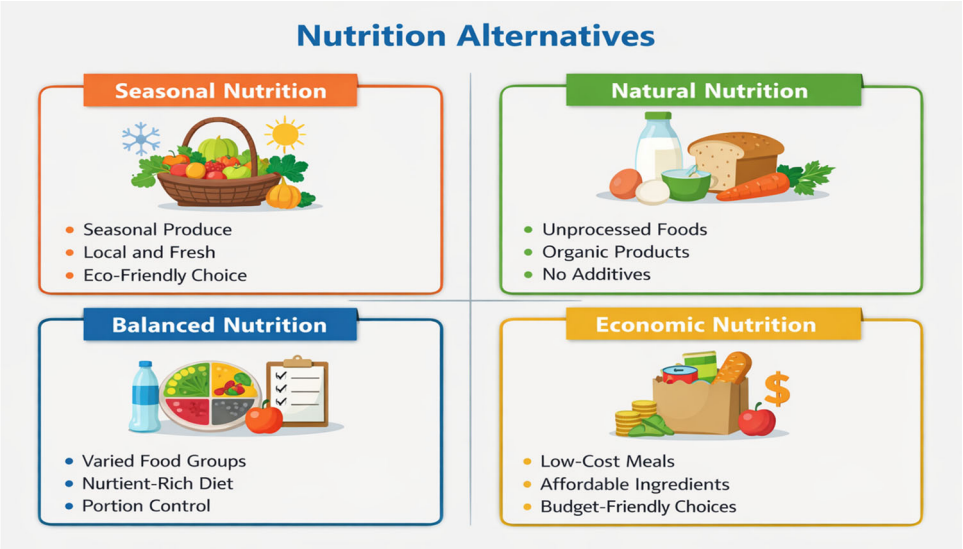


Fig. 4 Sustainable nutrition alternatives in early childhood

Table 3 Nutrition alternatives and criteria

A-Seasonal Nutrition	A1-Consuming fresh vegetables and fruits in season
	A2-Preferring foods that are bought in season and stored under appropriate conditions (frozen, dried, canned)
	A3-Not preferring imported products
B-Natural Nutrition	B1-Not preferring packaged foods containing preservatives
	B2-Do not prefer processed foods (fast food, etc.)
	B3-Preferring home-cooked meals
C-Adequate and Balanced Nutrition	C1-Consuming enough animal foods
	C2-Eating enough meals on time
	C3-Consuming enough plant foods
D-Economic Nutrition	D1-Not shopping for more food than needed
	D2-Not leaving portions unfinished
	D3-Not preferring single-use kitchenware (plates, spoons, forks, etc.)

preschool teacher is an expert with 21 years of experience (Table 3).

Step 2: The linguistic variables of DEs are represented in Table 4. Then, the linguistic variables are converted into FF numbers.

Step 3: The aggregated DEMA (Table 5) based on FF-numbers by Eq. 4 (or 5) is created. DEs weights are $\omega = \{0.15, 0.18, 0.20, 0.22, 0.35\}$.

Step 4: Obtain the entropy. $\mathscr{E}_A = 0.345$, $\mathscr{E}_B = 0.251$, $\mathscr{E}_C = 0.337$, $\mathscr{E}_C = 0.41$.

Step 5: Using the Eq. 8. Then obtain the weights of all criteria: $\omega_A = 0.255$, $\omega_B = 0.28$, $\omega_C = 0.245$, and $\omega_D = 0.22$.

Step 6: Determine the weighted DEMA (Table 6).

Step 7: Compute the score function (Table 7).

Step 8: Then, maximizing and minimizing indexes are

$$\overline{SC}(P_1) = 0.4004, \quad \overline{SC}(P_2) = 0.457,$$
$$\overline{SC}(P_3) = 0.4364, \quad \overline{SC}(P_4) = 0.357;$$
$$\overline{SC}(R_1) = 0.31, \quad \overline{SC}(R_2) = 0.313,$$
$$\overline{SC}(R_3) = 0.385, \quad \overline{SC}(R_4) = 0.49.$$

Step 9: The relative weight of each alternative are

$$Q_1 = 0.84, \quad Q_2 = 0.89, \quad Q_3 = 0.79, \quad Q_{14} = 0.64.$$

Step 10: The priority orders are

$$P_A = 94.38, \quad P_B = 100, \quad P_C = 88.76, \quad P_D = 71.91.$$

Table 4 IVFFNs of linguistic values given by DEs

	A	*B*	*C*	*D*
A_1	(EH, EH, EH, EH, EH)	(VH, H, EH, H, H)	(E, VH, AA, H, E)	(H, BA, E, AA, L)
A_2	(VH, EH, H, AA, E)	(H, EH, H, H, BA)	(E, H, H, E, L)	(VH, E, AA, H, AA)
A_3	(VH, VH, AA, H, E)	(H, E, H, H, BA)	(VH, E, H, VH, VL)	(VH, H, VH, VH, H)
B_1	(H, H, E, E, L)	(EH, EH, VH, EH, EH)	(VH, BA, H, L, E)	(E, L, E, VH, L)
B_2	(H, H, E, E, VL)	(EH, EH, VH, EH, VH)	(VH, BA, AA, H, H)	(E, L, BA, VH, L)
B_3	(E, E, VH, H, E)	(EH, EH, VH, VH, VH)	(VH, BA, H, AA, E)	(VH, AA, E, VH, E)
C_1	(VL, VL, L, L, EL)	(E, H, L, VL, H)	(EH, EH, EH, EH, EH)	(L, VL, E, AA, VL)
C_2	(E, VL, E, E, EL)	(H, L, EH, VH, EH)	(EH, EH, EH, EH, EH)	(E, VL, VL, E, VL)
C_3	(H, VL, H, AA, E)	(H, H, VL, BA, E)	(EH, EH, EH, EH, EH)	(E, VL, AA, BA, E)
D_1	(L, EL, H, BA, VL)	(L, EL, VL, VL, L)	(VH, L, E, AA, E)	(EH, EH, EH, EH, EH)
D_2	(L, EL, VL, VL, VL)	(L, EL, VL, L, VL)	(VH, L, VH, H, L)	(EH, EH, VH, VH, EH)
D_3	(EL, EL, VL, EL, EL)	(EL, EL, EL, EL, EL)	(E, H, VH, AA, H)	(EH, EH, EH, H, H)

Table 5 Aggregated DEMA

	A	*B*	*C*	*D*
A_1	(0.95, 0.15)	(0.838, 0.281)	(0.704, 0.415)	(0.580, 0.548)
A_2	(0.800, 0.338)	(0.792, 0.351)	(0.630, 0.501)	(0.709, 0.380)
A_3	(0.749, 0.369)	(0.676, 0.443)	(0.732, 0.418)	(0.815, 0.289)
B_1	(0.617, 0.512)	(0.938, 0.166)	(0.652, 0.493)	(0.633, 0.528)
B_2	(0.613, 0.528)	(0.920, 0.189)	(0.726 0.391)	(0.622, 0.546)
B_3	(0.698, 0.425)	(0.897, 0.211)	(0.684, 0.440)	(0.730, 0.396)
C_1	(0.290, 0.830)	(0.622, 0.530)	(0.95, 0.15)	(0.484, 0.665)
C_2	(0.467, 0.682)	(0.884, 0.255)	(0.95, 0.15)	(0.420, 0.724)
C_3	(0.644, 0.486)	(0.605, 0.536)	(0.95, 0.15)	(0.530, 0.593)
D_1	(0.510, 0.672)	(0.700, 0.493)	(0.640, 0.494)	(0.95, 0.15)
D_2	(0.260, 0.851)	(0.287, 0.828)	(0.722, 0.432)	(0.897, 0.211)
D_3	(0.180, 0.930)	(0.15, 0.95)	(0.706, 0.425)	(0.898, 0.223)

Table 6 DEMA with the weights of all criteria

	A	*B*	*C*	*D*
A_1	(0.730, 0.620)	(0.604, 0.701)	(0.464, 0.806)	(0.360, 0.876)
A_2	(0.551, 0.758)	(0.560, 0.746)	(0.408, 0.844)	(0.452, 0.808)
A_3	(0.506, 0.776)	(0.461, 0.796)	(0.486, 0.808)	(0.540, 0.761)
B_1	(0.404, 0.849)	(0.728, 0.605)	(0.424, 0.841)	(0.400, 0.869)
B_2	(0.401, 0.528)	(0.701, 0.627)	(0.481 0.794)	(0.389, 0.875)
B_3	(0.465, 0.804)	(0.670, 0.647)	(0.448, 0.818)	(0.468, 0.816)
C_1	(0.184, 0.954)	(0.420, 0.837)	(0.724, 0.628)	(0.297, 0.914)
C_2	(0.300, 0.907)	(0.654, 0.682)	(0.724, 0.628)	(0.256, 0.931)
C_3	(0.424, 0.832)	(0.408, 0.840)	(0.724, 0.628)	(0.327, 0.891)
D_1	(0.330, 0.904)	(0.481, 0.820)	(0.416, 0.841)	(0.704, 0.659)
D_2	(0.165, 0.960)	(0.188, 0.949)	(0.478, 0.814)	(0.626, 0.710)
D_3	(0.114, 0.982)	(0.100, 0.986)	(0.465, 0.811)	(0.627, 0.719)

Table 7 Score values

	A	*B*	*C*	*D*
A_1	0.538	0.469	0.400	0.344
A_2	0.433	0.440	0.367	0.400
A_3	0.416	0.400	0.397	0.430
B_1	0.363	0.541	0.370	0.352
B_2	0.480	0.524	0.403	0.347
B_3	0.395	0.507	0.392	0.390
C_1	0.284	0.372	0.533	0.316
C_2	0.320	0.491	0.533	0.302
C_3	0.375	0.369	0.533	0.332
D_1	0.324	0.390	0.370	0.516
D_2	0.280	0.288	0.392	0.472
D_3	0.324	0.260	0.392	0.469

Step 11: Hence the rank of the alternatives is $B > A > C > D$ (Fig. 5).

6.1 Sensitivity analysis

To assess the robustness of the proposed integrated decision-making framework, we conducted a sensitivity analysis by systematically varying the weights of the decision experts. The original expert weight vector is $\omega = \{0.15, 0.18, 0.20, 0.22, 0.35\}$. We generated ten perturbed weight vectors by randomly adding or subtracting up to 20% of the original weights and re-normalizing them to sum to one. For each perturbed vector, the entire decision-making process (Steps 1–11) was re-executed. The resulting priority orders for the four alternatives are summarized in Table 8.

The results demonstrate that the ranking $B > A > C > D$ remains stable across all scenarios, confirming the robustness of the proposed method and its insensitivity to minor variations in expert weight assignments.

7 Discussion

The given alternative criteria structure and the resulting $B > A > C > D$ ranking have produced a theoretically consistent, pedagogically meaningful, and policy- and practice-defensible result for evaluating sustainability-based feeding behaviors in early childhood (Fig. 6).

When examining the conceptual consistency of the Criterion-Alternative structure, we can look at the sustainability dimensions of the alternatives. The four selected alternatives represent different but complementary dimensions of sustainability: A-Seasonal Feeding: Environmental sustainability (carbon footprint, local production); B-Natural Feeding: Health-based sustainability + long-term behavioral persistence; C-Adequate and Balanced Nutrition: Biological/physiological sustainability; D-Economical Nutrition: Resource efficiency and consumption sustainability. This distinction shows that you are addressing sustainability in the context of early childhood not only as an environmental concept but also as a multidimensional one. In this respect, your model is strong.

When we examine the main reasons why Natural Nutrition (B) came in first place, this result reflects the strength of the B alternative criteria. The B1–B3 criteria have the following characteristics: They are behaviorally clear and observable, directly applicable in family and school environments, and contribute most to habit formation in early childhood. Specifically, avoiding processed foods, preferring home-prepared meals, and excluding products containing additives ensures the internalization of healthy eating norms at an early age. These norms also form the basis of environmentally and economically sustainable behaviors in later life. Therefore, Natural Nutrition's first-place ranking can be explained not only by current sustainability indicators but also by its capacity to generate behaviors that can be carried into the future.

Let us examine why seasonal eating (A) ranks second. The fact that alternative (A) comes in second is an expected and rational result, because criteria A1 and A2 are strong in terms of sustainability. However, A3 (avoiding imported products) is not directly under the child's control in early childhood; it depends on family decisions, and its applicability is more sensitive to socioeconomic conditions. This situation may have led to alternative (A), despite its environmental strength, having a more limited level of behavioral transformation in early childhood compared to (B).

The reason why adequate and balanced nutrition is ranked third (C) can be explained as follows: Criteria (C) (C1–C3) are essential for health, but the sustainability aspect is indirect. For example, adequate consumption of animal products is positive from a nutritional standpoint. However, it is debatable in terms of environmental sustainability. Meal planning ensures health sustainability, but the environmental/economic impact is limited. Therefore, the placement of alternative (C) in a "fundamental but non-transformative" position is consistent with the logic of the analysis.

Let us interpret the fact that economical eating (D) is ranked last. Criteria D1–D3 are directly related to reducing waste, resource efficiency, and consumer awareness. However, in early childhood, portion control, shopping quantity, and preference for single-use products are influenced mainly by parental behaviors, relying more on indirect learning than on the child's individual preferences. Therefore, the fact that alternative (D) is ranked last does not mean that the criteria are unimportant; rather, it indicates a lower potential for developing direct behaviors in early childhood.

Let us examine the overall assessment and academic strengths. Examining the strengths of the results, the rank-

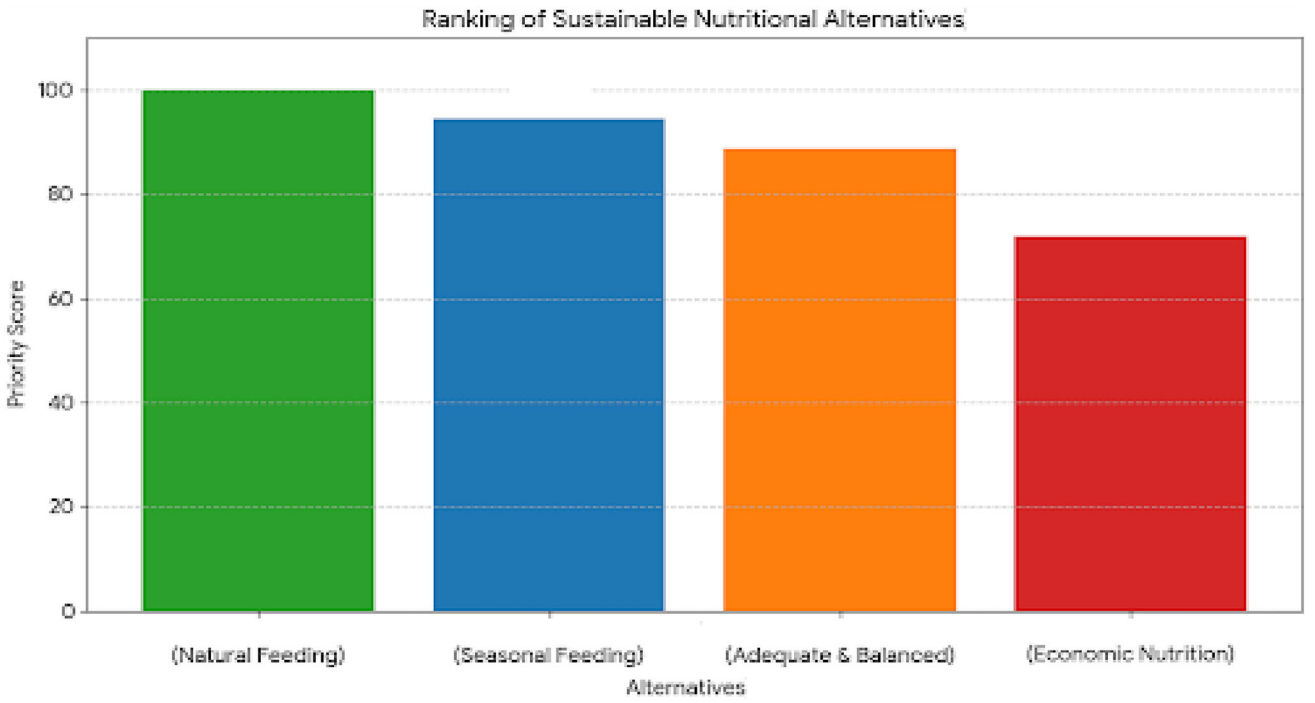


Fig. 5 Ranking of sustainable nutritional alternatives

Table 8 Priority orders under perturbed expert weights

Scenario	Alternative A	Alternative B	Alternative C	Alternative D
Original	94.38	100.00	88.76	71.91
Scenario 1	93.12	100.00	87.45	72.33
Scenario 2	94.89	100.00	89.01	71.02
Scenario 3	95.01	100.00	88.12	70.88
Scenario 4	92.78	100.00	88.99	73.11
Scenario 5	94.15	100.00	89.54	71.54
Scenario 6	93.55	100.00	87.76	72.89
Scenario 7	94.44	100.00	88.33	71.45
Scenario 8	95.23	100.00	88.67	70.45
Scenario 9	93.89	100.00	89.22	71.99
Scenario 10	94.66	100.00	87.99	71.66

ing is consistent with pedagogical reality, a causal hierarchy exists among the alternatives, and natural nutrition serves as a precursor to other alternatives. Academically, we can say: "The results show that natural nutrition-focused practices play a more decisive role in establishing sustainable nutritional behaviors in early childhood compared to seasonality, balanced nutrition, and economic consumption approaches."

If we evaluate the study from a behavioral sustainability perspective, we can conclude the following: The findings reveal that natural nutrition-focused practices (B) play a central role in establishing sustainable eating behaviors in early childhood and demonstrate a high degree of consistency with the existing literature. Previous studies emphasize that avoiding processed and additive-containing foods, preferring home-cooked meals, and opting for natural foods are crucial in developing healthy and sustainable eating habits at an early age. These behaviors are not limited to short-term health outcomes; they also create a strong behavioral foundation for individuals to develop environmental awareness, conscious consumption, and responsible food choices in later life stages.

The fact that the alternative order is $B > A > C > D$ can also be explained by behavioral sustainability models. These models suggest that the fundamental element determining the permanence of sustainable behaviors is the direct, repeatable, and internalized behaviors acquired by the individual at an early age. In this context, natural nutrition offers a set of behaviors that a child can easily experience in their



Fig. 6 Ranking of sustainable nutritional alternatives

daily life practices and that can be quickly learned through parent–child interaction. Seasonal nutrition (A) is a strong alternative in terms of environmental sustainability; however, its level of transformation into behavior is relatively limited, as the decision-making mechanism largely depends on adults. Adequate and balanced nutrition (C) is a fundamental requirement supporting physiological sustainability, but it is more indirectly related to environmental and economic sustainability dimensions. The fact that economical nutrition (D) is ranked last can be explained by the fact that these behaviors are based on parental control rather than direct individual choices in early childhood. Therefore, the resulting ranking shows that sustainability in early childhood is primarily built through nutritional practices with a high capacity for behavioral internalization.

7.1 Behavioral sustainability model

This subsection provides the theoretical framework for explaining why the alternate ordering occurs in this manner.

Table 9 Comparison of criterion weights

Criterion	Proposed FF-entropy	CRITIC
A (Seasonal)	0.255	0.248
B (Natural)	0.280	0.291
C (Balanced)	0.245	0.239
D (Economic)	0.220	0.222

This study proposes a "Behavioral Sustainability-Based Nutrition Model" to explain sustainable eating behaviors in early childhood. The model is based on the assumption that sustainability develops in early childhood not through environmental or economic awareness, but through "habits internalized through repeatable daily behaviors."

The model consists of three main layers. The first layer represents "directly experiential behaviors" and encompasses nutritional practices that a child can actively observe and apply in their daily life. Natural eating (B) is included in this layer and forms the behavioral basis of sustainability through behaviors such as avoiding processed foods and consuming home-prepared foods.

The second layer includes "behaviors requiring contextual and environmental awareness." Seasonal eating (A), while strong in terms of environmental sustainability, requires indirect knowledge about the production-consumption chain; therefore, it can only be translated into behavior to a limited extent in early childhood.

The third layer represents "behaviors based on indirect learning." Adequate and balanced nutrition (C) and economical nutrition (D) are behaviors shaped mainly by parental control and adult guidance, and have a limited impact on the child's individual preferences and decision-making mechanisms.

This layered structure explains the order of alternatives as $B > A > C > D$, through the capacity for internalization and sustainability of these behaviors at an early age.

7.2 Comparative analysis

To further validate the effectiveness of the proposed integrated approach, we conducted a comparative analysis against other well-established MCDM methods under the same Fermatean fuzzy environment.

7.2.1 Comparison of weighting schemes

The objective weights obtained by the proposed FF-entropy measure are compared with those derived from the CRITIC method, as presented in Table 9. The CRITIC method determines weights based on the contrast intensity and conflict among criteria.

The weights obtained from both methods are highly consistent, with natural nutrition (B) receiving the highest weight and economic nutrition (D) the lowest. This consistency reinforces the validity of the proposed entropy-based weighting scheme.

7.2.2 Comparison of ranking methods

The ranking of alternatives obtained by the proposed FF-entropy-COPRAS method is compared with those from FF-TOPSIS and FF-VIKOR using the same weights ($\omega_A = 0.255, \omega_B = 0.28, \omega_C = 0.245, \omega_D = 0.22$). The comparative rankings are shown in Table 10.

The results show that the ranking order ($B > A > C > D$) is identical across all three methods. This agreement demonstrates the robustness and reliability of the proposed integrated approach, confirming that the findings are not an artifact of a specific MCDM technique.

7.3 Implications

Policy Implications: The findings of this study suggest that preschool education policies should be restructured within the context of sustainability. The fact that natural nutrition (B) ranks first suggests that sustainable nutrition education should primarily be addressed in early childhood through a "behavior-based and practice-oriented" approach. In this context, it is recommended that preschool education programs include more activities promoting home-cooked meals, unprocessed foods, and additive-free nutrition practices that encourage children's active participation.

The fact that seasonal nutrition (A) ranks second reveals that environmental sustainability awareness can be instilled at an early age, but this should be supported through "concrete experiences" rather than abstract concepts. Practices such as vegetable-growing activities in school gardens, seasonal calendars, and local food presentations can facilitate the transformation of this awareness into action.

The lower ranking of adequate and balanced nutrition (C) and economical nutrition (D) does not mean that these areas should be neglected in education policies; on the contrary, it shows that these behaviors should be supported in early childhood through "parental involvement and family-school cooperation". In this context, parent education programs, family guides, and home-based sustainable nutrition practices should be considered as complementary elements of preschool policies.

In conclusion, this study reveals that structuring sustainable nutrition in preschool education policies based on "behavioral internalization and daily practices" rather than cognitive knowledge transfer will produce more effective and lasting outcomes.

Managerial implications: The findings of this study demonstrate that managerial decisions play a critical role in developing sustainable nutritional behaviors in early childhood. The ranking of alternatives as "Natural Nutrition > Seasonal Nutrition > Adequate and Balanced Nutrition > Economical Nutrition" provides a prioritized roadmap for preschool administrators and policymakers implementing educational policies.

Firstly, the prioritization of natural nutrition suggests that nutritional policies in preschools should be structured to systematically limit processed and additive-containing foods. Institution administrators should encourage home-made and natural-ingredient menus in cafeteria and canteen arrangements, and prioritize additive-free, local, and reliable products in supplier selection. This approach will support not only short-term health outcomes but also the establishment of long-term sustainable nutritional behaviors at the institutional level.

The second ranking of seasonal nutrition highlights the importance of "seasonality-oriented menu design" in managerial planning. Institution administrators should organize annual and seasonal menu plans in line with seasonal calendars, and strengthen cooperation with local producers by reducing the use of imported products. Such practices will support environmental sustainability while also contributing to children's development of seasonal awareness.

The fact that adequate and balanced nutrition ranks third indicates that this area should be supported through internal standards and guidelines. Managers should plan balanced menus based on food groups; however, these practices should be implemented with flexible structures that are adaptable to children's ages and developmental characteristics, rather than rigid rules. In this way, nutritional sustainability will be supported without creating behavioral pressure.

The fact that economical nutrition ranks last suggests that waste reduction and resource efficiency practices should be addressed through "institutional management processes" rather than directly through children. Practices such as portion control, tracking food waste, and reducing single-use products can be implemented more effectively through decisions made at the management level.

In general, these findings reveal that sustainable nutrition management in preschools should primarily be built on "natural nutrition practices that directly teach behavioral changes." In contrast, a phased and holistic management approach should support the environmental and economic sustainability dimensions.

7.4 Limitations

While this study provides valuable insights, it is not without limitations. First and foremost, the entire decision-making process relies on the subjective judgments of a limited

Table 10 Comparison of alternative rankings

Alternative	Proposed COPRAS	FF-TOPSIS	FF-VIKOR
A (Seasonal)	2	2	2
B (Natural)	1	1	1
C (Balanced)	3	3	3
D (Economic)	4	4	4

number of experts. Although these experts are highly experienced, their perspectives may not fully capture the diversity of opinions across different cultural, socioeconomic, or geographical contexts. This reliance on expert opinion, rather than on direct observational or experimental data, constitutes a primary limitation of the study.

Second, the analysis is conducted within a static decision-making framework. However, nutritional behaviors in early childhood are dynamic and evolve over time. The model does not account for these temporal dynamics or the changing priorities that may emerge as children grow.

Third, the scope of the criteria and alternatives, while grounded in the literature, may not be exhaustive. Cultural differences, regional dietary habits, and individual family dynamics are complex variables that were not directly integrated into the model. This may limit the direct transferability of the results to other settings.

Finally, the study employs a single methodological framework (FF-entropy + COPRAS). Although we included a comparative analysis with other methods, the results are inherently tied to this specific approach. Exploring the problem with different fuzzy extensions or MCDM structures could yield different insights.

8 Conclusion

This study examines early childhood nutritional behaviors from a sustainability perspective and models them as a multi-criteria decision problem involving uncertainty. The main objective of the study is to evaluate different sustainability approaches to early childhood nutrition holistically and to establish a rational prioritization among them. The COPRAS approach, developed in this context and based on Fermatean fuzzy entropy, effectively reflects the uncertainty in expert opinions and provides an analytical basis for comparing sustainable nutrition alternatives.

The results show that natural nutrition is the most prioritized alternative for developing sustainable nutritional behaviors in early childhood. Criteria such as avoiding processed foods, preferring additive-free products, and preparing home-cooked meals strongly contribute to the formation of healthy, lasting nutritional habits at an early age. This finding reveals that natural nutrition plays a critical role not only in current health benefits but also in shaping behavioral patterns that can be carried forward.

Seasonal feeding, ranked second, is a strong alternative for environmental sustainability, but its behavioral transformation in early childhood largely depends on parental decisions. Adequate and balanced nutrition, while essential from a biological and physiological perspective, ranked third due to its more indirect relationship with the environmental and behavioral dimensions of sustainability. The last-place ranking of economic feeding stems not from the approach's insignificance, but from its limited potential for direct behavioral development in early childhood.

Overall, the study shows that sustainable eating behaviors in early childhood should be considered not only for their nutritional content but also for their learnability, continuity, and lifelong transferability. The proposed method provides a robust analytical framework that can serve as a decision-support tool for policymakers, educators, and health professionals in early childhood nutrition. Future studies are recommended to test the model with larger samples, to integrate cultural and socioeconomic variables, and to examine different fuzzy structures comparatively.

Future research: Future studies can be developed in several directions to expand and deepen the findings of this research. First, strengthening the group decision-making structure by involving a larger number of experts from different disciplines will increase the reliability and generalizability of the model's results. Furthermore, multi-stakeholder decision-making models involving parents, teachers, and health professionals can be developed.

Secondly, applying the model in different cultural, geographical, and socioeconomic contexts will contribute to a better understanding of the context-specific dynamics of sustainable feeding behaviors. In this context, cross-country comparative analyses or regional applications offer an important research area for future studies.

Thirdly, integrating dynamic, time-sensitive decision-making approaches into the model can better reflect changes in early childhood feeding behaviors across development. Dynamic MCDM models or longitudinal data structures can be considered in this regard.

Methodologically, future studies can focus on the comparative analysis of different fuzzy structures (e.g., heuristic, Pythagorean, or interval-valued Fermatean fuzzy sets) and

different criterion weighting or ranking methods. Such comparisons will more clearly reveal the methodological advantages of the proposed approach.

Finally, supporting expert-based decision-making outcomes with observational, experimental, or intervention-based studies is important for evaluating the model's impact in real-life applications. Empirical studies through school-based nutrition programs or parent education interventions will provide stronger evidence on how sustainable nutritional behaviors can be developed in early childhood.

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Declarations

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